Interacting Information Cascades: On the Movement of Conventions Between Groups*

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Abstract

When a decision maker is a member of multiple social groups, her actions may cause information to "spill over" from one group to another. We study the nature of these spillovers in an observational learning game where two groups interact via a common player, and where conventions emerge when players follow the decisions of the members of their own groups rather than their own private information. We show that: (i) if a convention develops in one group but not the other group, then the convention spills over via the common player; (ii) when conventions disagree, then the common player's decision breaks the convention in one group; and (iii) when no convention has developed, then the common player's decision triggers the same convention in both groups. We also show that information spillovers may reduce welfare and we investigate the surplus-maximizing timing of spillovers.

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1 Introduction

Decision makers are typically members of multiple social groups. Consider, for example, an economics professor. She belongs to an economics department and may belong to a group of faculty who regularly get together to play poker. As a member of these groups, she is a conduit through which information flows, not only within a group, but also from one group to another. She may learn, for example, from fellow poker players about university health plans, and she may transmit this information, via her choice of plan, to her colleagues in the economics department. Similarly, information flows both within and between the divisions of a firm through its senior managers. Information is also conveyed within and between different groups of friends on social networking sites, like Facebook, via friends, friends of friends, and so on, who are members of multiple groups.

Our goal is to better understand when these spillovers occur, their behavioral effects, and their welfare consequences. To do this, we extend Bikhchandani et al. (1992) and study "interacting cascades," in which two different groups of players share a common player. Each player chooses whether to adopt or reject a behavior, where it is optimal to adopt if the true (but unknown) state is high and reject if the state is low. Prior to making her decision, each regular player observes an informative private signal as well as the prior decisions of the members of her own group. The common player observes her own private signal and the decisions of her predecessors in both groups. Since players moving after the common player observe her decision, these players indirectly learn about the decisions and information of the players in the other group. In this way, the common player allows information to flow between groups and allows behavior in one group to influence behavior in the other.

For players moving prior to the common player, an interacting cascade is identical in structure to Bikhchandani et al.'s (1992) model of an information cascade and has the same equilibrium structure: a player follows her own signal, choosing adopt if her signal is high and reject if her signal is low, so long as the difference in the number of adopt and reject decisions by prior players does not exceed one. If, however, the number of adopt (reject) decisions exceeds the number of reject (adopt) decisions by two or more, then a player ignores her own signal and also chooses adopt (reject), as the information revealed by the decisions of prior players outweighs the information contained in her own private signal. The player is then said to be in a "cascade" on adopt (reject).

Our focus is on the decisions of the common player and the players who move after her. We show that if an information cascade forms in only one group prior to the move of the common player, then the cascade spills over via the common player to the other group. If, however, the groups are in cascades on different decisions, then the common player breaks the cascade in one group. In this case, the common player follows her own signal, the cascade that agrees with the common player's decision continues, and the cascade that disagrees with the common player's decision ends. Finally, if neither group is in a cascade, then the common player follows her own signal and triggers cascades (on her decision) in both groups.

Surprisingly, information spillovers via a common player need not enhance welfare. Intuitively, this occurs because the common player suppresses learning when her decision triggers cascades in both groups. Nevertheless, we provide sufficient conditions for information spillovers to be welfare enhancing. In particular, the payoff of every player is weakly higher in an interacting cascade if either four or more players in each group move prior to the common player or the private signals are sufficiently informative.

The common player's position has implications for welfare – if her turn to move comes too early, then she shuts down learning prematurely, while if it comes too late, then fewer players benefit from her aggregation of information. The optimal position of the common player balances these two effects. We show that if the signal precision is high (and thus the benefit from extending information aggregation is small), then total surplus is maximized when the common player's turn to move comes early, while if it is low then surplus is maximized when the common player moves late.

Our results extend information cascades in a way first suggested by Bikhchandani et al. (1992) and shed light on the how the structure of social networks affects the adoption of products and behaviors. Lindstrom and Muñoz-Franco's (2005) study of contraceptive use illustrates our result that if a cascade emerges in one group but not the other, then the cascade spills over. The authors find that urban migrants (the common players) transmit the urban convention of contraceptive use back to their rural villages, where its use is sporadic. Javorcik's (2004) study of spillovers from foreign direct investment provides another example of this result. He finds that ventures by foreign firms from developed countries (the common players) transmit productivity-enhancing practices to their up-stream suppliers in developing countries and those suppliers' domestic competitors. Rogers (1983) documents a third example where the adoption of solar water heaters spills over from one group of homeowners to another via common acquaintances.

The balance of this section discusses the related literature. Section 2 introduces our model. Section 3 states our results and Section 4 concludes. The Appendix contains the omitted proofs and a detailed characterization of equilibrium.

Related Literature

Our work builds on the seminal observational learning models of Banerjee (1992) and Bikhchandani et al. (1992).[1] These papers, as well as much of the subsequent observation

[1] Experimental and empirical studies have demonstrated the empirical relevance of observational learning.

learning literature (e.g., Lobel and Sadler (2014), Muller-Frank (2014), and Wu (2015)), focus on whether players asymptotically learn the true state as the group grows large – and thus make optimal decisions – under different signaling structures.[2] Our focus is on small groups where limit results do not apply. We examine the interaction of cascades, the welfare consequence of information spillovers between groups, and the optimal timing of information spillovers. While the question of whether players learn the true state as their number grows large is important since it speaks to long-run welfare, it is also important to understand behavior in small groups.

Nonetheless, there are several related results in the literature. Bikhchandani et al. (1992) examines the case where one player in an information cascade has a more accurate signal than the rest. It shows that this player can break a cascade when her signal is sufficiently precise. While there is a parallel between this player and our common player, there is also an important difference: the informational advantage this player enjoys relative to the other players is exogenous, whereas the informational advantage of our common player is endogenous. More importantly, in Bikhchandani et al.'s framework, there is only a single group of players, and thus there is no scope for the spillovers between groups, on which we focus. Another study relevant for small groups is Goeree et al. (2007), which shows that cascades may break when players' decisions are noisy, as in a Quantal Response equilibrium.

Our environment resembles Example 4 in Lobel and Sadler (2014), which one can interperet as having two groups that share an infinate sequence of common players. The authors show that asymptotic learning does not obtain. Our simpler setting allows us to completely characterize equilibrium in order to address information spillovers.

Our results are related to those of Cipriani and Guarino (2008). In a setting quite different from the Bikhchandani et al. and Banerjee framework, the authors examine markets for two assets, whose fundamental values are correlated, and show that (i) information may spillover from one market and give rise to a cascade in a second market and (ii) that price movement in the second market may break the cascade in the first market. Whereas Cipriani and Guarino allow all traders to observe the entire history of asset prices in both markets, in our setting players observe their own group's histories and the common player's decision. This leads to differences in our results – e.g., Cipriani and Guarino show that learning stops forever once both markets are in cascades, while we find that if both groups are in different cascades, then learning restarts.

See Anderson and Holt (1997), Drehmann et al. (2005), Cai et al. (2009), and Weizsacker (2010), among others.

[2]Key papers in the observational learning literature include Acemoglu et al. (2011), Banerjee and Fudenberg (2004), Cao et al. (2011), Callander and Horner (2009), Celen and Kariv (2001), Golub and Jackson (2010), Guarnio et al. (2011), Guarino and Jehiel (2009), and Smith and Sorensen (2000).

2 The Model

This section describes the environment and the solution concept.

Basic Cascades

In the basic cascade introduced in Bikhchandani et al. (1992), N identical players move sequentially in a commonly known and exogenous order. Let i denote the i-th player to move, where $i \in \{1, 2, \ldots, N\}$. When it is her move, a player decides whether to adopt (a) or reject (r) a behavior. A player's payoff depends only on her own decision and the true, but unknown state, which may be either high ($\mathcal{H}$) and low ($\mathcal{L}$). Her payoff is 1 if she adopts in state $\mathcal{H}$, -1 if she adopts in state $\mathcal{L}$, and 0 if she rejects. Formally, the payoff of player i when she makes decision $d_i \in \{a, r\}$ in state $s \in \{\mathcal{H}, \mathcal{L}\}$ is

$$u(d_i, s) = \begin{cases} 1 & \text{if } d_i = a \text{ and } s = \mathcal{H} \\ -1 & \text{if } d_i = a \text{ and } s = \mathcal{L} \\ 0 & \text{if } d_i = r. \end{cases}$$

Each state is equally likely, i.e., $P(\mathcal{H}) = P(\mathcal{L}) = 1/2$.[3] Each player i observes the decisions of the prior players $\bar{d}_{i-1} := (d_1, \ldots, d_{i-1})$ and an informative private signal $x_i \in \{H, L\}$, prior to making her own decision. The probability of signal x, conditional on the true state being s, is

$$P(x|s) = \begin{cases} p & \text{if } x = s \\ 1-p & \text{if } x \neq s, \end{cases}$$

where $p \in (1/2, 1)$. Given s, the signals x_i and x_j are independent when $i \neq j$. We write $\bar{x}_i$ for $(x_1, \ldots, x_i)$.

Interacting Cascades

We study interacting cascades in which there are two groups – A and B – each with N players. In each group, the players move sequentially in a commonly known and exogenous order. We write i^g for the i-th player to move in group $g \in \{A, B\}$. The groups interact via a common player who is a member of both groups and who is the k-th player to move in each group. For simplicity, we take k to be odd and $1 < k < N$.[4] The arrangement of the players is illustrated in Figure 1. Each player observes the decisions of the prior players in her own group and an informative private signal prior to making her own decision. The common player observes the decisions of the prior players in *both* groups, in addition to an informative private signal.

[3] We discuss asymmetric priors in the Conclusion and provide a full characterization of equilibrium for this case in Fisher and Wooders (2016).

[4] We discuss the case where k is even in the Conclusion.

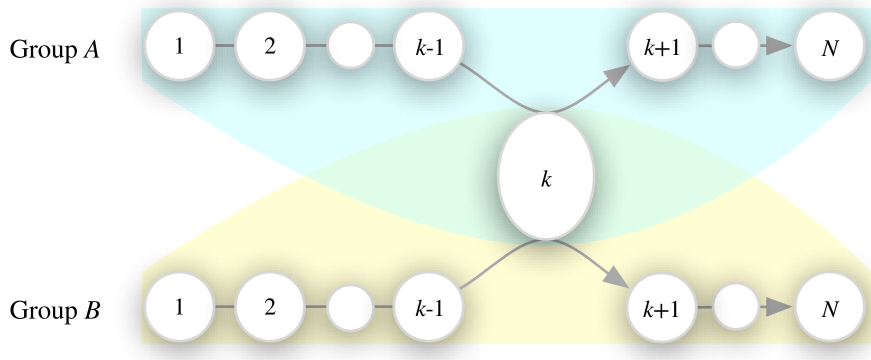

Figure 1: An Interacting Cascade

We write x_i^g and d_i^g for the signal and decision of the i-th player in group g, respectively, and we write d_k^A, d_k^B, or simply d_k for the decision of the common player. We write $\bar{d}_i^g$ for $(d_1^g, \ldots, d_i^g)$ for the decisions of players 1 through i in group g and $\bar{x}_i^g$ for $(x_1^g, \ldots, x_i^g)$ for the signals of 1 through i in group g.

Our solution concept is Perfect Bayesian Equilibrium. We focus on the equilibrium in which players "follow their signals" when indifferent. It is easy to see, via an induction argument, that such an equilibrium exists and is unique.[5] In equilibrium, after player i^g observes the history $\bar{d}_{i-1}^g$ and her signal x_i^g, she forms a belief ϕ, according to Bayes' Rule, that the state is $\mathcal{H}$. Since her expected payoff to a is $\phi u(a, \mathcal{H}) - (1-\phi)u(a, \mathcal{L}) = 2\phi - 1$ and her expected payoff to r is 0, she chooses a if $\phi > 1/2$ and r if $\phi < 1/2$. If $\phi = 1/2$, she follows her signal by choosing a if $x_i^g = H$ and r if $x_i^g = L$.

We say that player i^g is in a **cascade on $\boldsymbol{a}$ $(\boldsymbol{r})$** if she chooses a (r) for any realization of her private signal. If a player is not in a cascade, then subsequent players in group g can infer her signal from her decision.

3 Results

In this section, we characterize equilibrium in interacting cascades. We also evaluate the welfare consequences of information spillovers and consider the welfare-maximizing placement of the common player.

[5] The argument is standard – e.g., Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992) – so we only sketch it. Player 1^g observes her signal, applies Bayes' Rule to determine her belief, and her strategy is uniquely determined. Player 2^g observes her signal and 1^g's decision, applies Bayes' Rule to determine her belief (as knowledge of 1^g's strategy allows her to determine the set of signals that 1^g could have received), and her strategy is uniquely determined. In general, a player uses the strategies of her predecessors to determine the signals they could have received and then employs Bayes' Rule to determine her belief. Hence, her strategy is also uniquely determined.

Prior to the common player, an interacting cascade is identical to the basic cascade introduced in Bikhchandani et al. (1992), who establish the following result.

Proposition 1. *In equilibrium, a player moving before the common player is in a cascade on a (r) if the number of a (r) decisions by her predecessors exceeds the number of r (a) decisions by two or more. Otherwise, she chooses a (r) given signal H (L).*

The intuition is that once the number of a decisions by a player's predecessors exceeds the number of r decisions by two or more, the information conveyed by the prior decisions that the state is $\mathcal{H}$ outweighs the information conveyed by the player's own private signal. Thus she is in a cascade on a. Since her decision is uninformative, all of her successors (who move before the common player) also choose a, regardless of their private signals. Hence, information aggregation ceases.

We write $w^B(i)$ for the payoff of the i-th player to move in a basic cascade. One can show that $w^B(i) = \frac{2p-1}{2}\Lambda(i+1)$ for i even and $w^B(i+1) = w^B(i)$ for odd i, where $\lambda = 2p(1-p)$ and

$$\Lambda(i) = \frac{1-\lambda^{\frac{i}{2}}}{1-\lambda}.$$

Players moving prior to the common player get the same payoff in an interacting cascade and a basic cascade.

We say that a *group* is **in a cascade on $\boldsymbol{a}$ ($\boldsymbol{r}$)** if the number of a (r) decisions by the first $k-1$ players in the group exceeds the number of r (a) decision by two or more. Proposition 2 identifies the equilibrium behavior of the common player.

Proposition 2. *In equilibrium, the common player is in a cascade on a (r) if either (i) both groups are in a cascade on a (r) or (ii) one group is in a cascade on a (r) and the other group is not in a cascade. Otherwise, the common player chooses a (r) given the signal H (L).*

The intuition is that, when (i) both groups are in a cascade on the same action or (ii) one group is in a cascade and the other group is not, then the information conveyed by the decisions of the common player's predecessors outweighs her private signal, so she follows the cascade. Otherwise, the information conveyed by the common player's predecessors is sufficiently ambiguous that she follows her signal.

Proposition 3 is our first main result. It identifies the conditions under which (i) a cascade "spills over" from one group to another, (ii) information spillovers end an existing cascade, and (iii) the common player's decision "triggers" a new cascade, i.e., begins a cascade when there was none before.

Proposition 3: *In equilibrium, if prior to the common player:*

P3.1 : Both groups are in cascades on a, then each cascade continues, i.e., the common player and all subsequent players in both groups choose a.

P3.2 : One group is in a cascade on a and the other group is not in a cascade, then the cascade on a "spills over" to the other group, i.e., the common player and all the subsequent players in both groups choose a.

P3.3 : Both groups are in cascades on different decisions, then (i) the common player follows her own signal, (ii) the cascade that agrees with the common player's decision continues, and (iii) the cascade that disagrees with the common player's decision ends, i.e., player $k+1$ in the group whose cascade disagrees with the common player's decision follows her own signal.
P3.4 : Neither group is in a cascade, then the common player "triggers" two cascades, i.e., the common player follows her own signal and all subsequent players in both groups make the same decision as the common player.
The analogous statements apply to cascades on r.

The intuition is that players moving after the common player in a group cannot tell how well informed the common player is based on her observation of the other group – she might be highly informed (e.g., sees many identical decisions) or minimally informed (e.g., sees many contradictory decisions). Nevertheless, her decision conveys some information about the state. For a group not in a cascade, the common player's decision is convincing enough to cause them to enter a cascade on it. For a group in a cascade, the decision is convincing enough to (i) cause them to doubt their cascade and follow their signals (for a time) when it disagrees with their cascade and (ii) cause their cascade to continue when it agrees with their cascade.

Proposition 4 gives each player's equilibrium payoff in an interacting cascade. The proof is computational and omitted.

Proposition 4. *In an interacting cascade, the equilibrium payoff of player i^g is*

$$w(i,k) = \begin{cases} \frac{2p-1}{2}\Lambda(i+1) & \text{for } i<k \text{ and } i \text{ odd} \\ \frac{2p-1}{2}\Lambda(2k) + (2p-1)[\frac{\lambda}{2}\Lambda(k-1)]^2 & \text{for } i=k \\ \frac{2p-1}{2}\Lambda(2k) + (2p-1)[\frac{\lambda}{2}\Lambda(k-1)]^2[1+\frac{\lambda}{2}\Lambda(i-k)] & \text{for } i>k \text{ and } i \text{ odd.} \end{cases}$$

Furthermore, the even player moving immediately after an odd player obtains the same payoff as the odd player, $w(i+1,k) = w(i,k)$ for i odd.

The equilibrium payoffs of players i^g and $i+1^g$ are the same when i is odd. For $i \neq k$, this is a consequence of the fact that i^g and $i+1^g$ are either (i) both in the same cascade or (ii) both follow their own signal. For $i = k$, this is a consequence of the fact that $k+1^g$

makes the same decision as k, unless both groups are in cascades on different actions, in which case each follows her own signal.

Since $\Lambda(i)$ is increasing, the following is an immediate corollary of Proposition 4.

Corollary 1. *Players positioned later in an interacting cascade obtain higher payoffs. More precisely, the payoff of every odd player is strictly greater than the payoff of the odd player who immediately precedes her.*

Let $c(i,k)$ denote the equilibrium probability that player i^g makes a correct decision, i.e., chooses a if the state is $\mathcal{H}$ and chooses r if the state is $\mathcal{L}$. Player i^g's payoff $w(i,k)$ can be written as $\frac{1}{2}c(i,k) + \frac{1}{2}[-(1-c(i,k))] = c(i,k) - 1/2$, i.e.,[6]

$$c(i,k) = w(i,k) + 1/2.$$

It follows that every result about i^g's equilibrium payoff is equivalent to a result about the probability she makes a correct decision – e.g., Corollary 1 implies that later players have a greater chance of making correct decisions.

The Welfare Consequences of Information Spillovers

Our next results compare player payoffs in interacting and basic cascades, and thus shed light on the costs and benefits of information spillovers between groups. From Proposition 4 it is clear that $w(k,k) > w^B(k)$, i.e., the common player in an interacting cascade obtains a higher payoff than her counterpart in a basic cascade. The intuition for this result is straightforward: The common player has better information. Since she observes both groups, she infers at least two more signals than her counterpart does in a basic cascade.

One might conjecture that the introduction of a common player raises the payoff of every player who moves subsequently relative to what that player would obtain in a basic cascade. This conjecture is not correct as the following example illustrates. Figure 2 (below) shows player payoffs as a function of position in an interacting cascade, with $k = 3$ and $p = 0.6$, and in a basic cascade, with $p = 0.6$.

In this example, players who move late in the basic cascade have higher payoffs than players in the same position in the interacting cascade. For player 11^g, for instance, $w^B(11) \approx 0.190 > w(11,3) \approx 0.188$, and every player who moves subsequently has a higher payoff in the basic cascade.

Player 11^g is affected in several ways (some good and others bad) by information spillovers through the common player. First, these spillovers may cause an existing cascade to end. If

[6] If the true state is H, with probability $c(i,k)$ player i^g chooses a and obtains 1 and with probability $1-c(i,k)$ chooses r and obtains 0. If the true state is L, player i^g obtains 0 with probability $c(i,k)$ and -1 with probability $1-c(i,k)$.

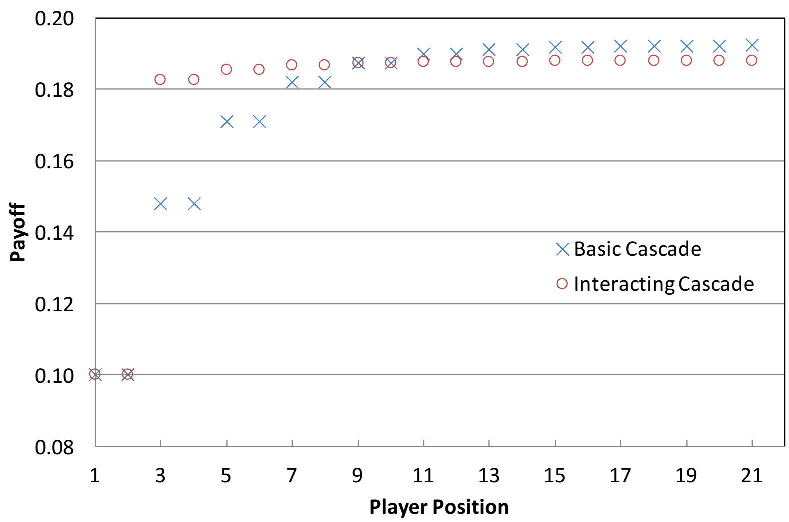

Figure 2: Payoffs by Position, $k = 3$ and $p = 0.6$

players 1^g and 2^g both make the correct decision, then in the basic cascade player 3^g and every subsequent player is in a cascade and makes the correct decision as well. In contrast, in the interacting cascade, if players 1^g and 2^g both make the correct decision, then player 11^g makes a correct decision only with probability 0.962. In particular, if the other group is in a cascade on the incorrect decision, the information spillover may end the correct cascade, thereby lowering player 11^g's payoff. Conversely, information spillovers may end an incorrect cascade. If players 1^g and 2^g both make the incorrect decision, then in the basic cascade every subsequent player makes the incorrect decision as well. In contrast, in an interacting cascade player 11^g makes the correct decision with probability 0.189.

A more subtle effect of information spillovers is their potential to suppress positive information externalities. Suppose that players 1^A and 2^A make opposing decisions. In the basic cascade, player 3^A follows her own signal and makes the correct decision with probability p ($= 0.6$ here). Player 11^A, however, makes the correct decision with a higher probability of .687. The difference between these probabilities is the positive information externality that player 11^A enjoys from observing the decisions of players 3^A through 10^A. In the interacting cascade, by contrast, when players 1^A and 2^A make opposing decisions, the players in group A are certain to be in an information cascade on the common player's decision, and thus enjoy no additional information externalities. (In particular, either a cascade in B spills over to group A (see P3.2) or the common player triggers a cascade in A (see P3.4).) The probability that player 11^A makes the correct decision is only 0.648. The strength of these effects depends on the signal accuracy and the location of the common player.

Proposition 5 is our second main result. It identifies conditions under which the common player aggregates information in a strong sense – she obtains a higher payoff than *every* player in a basic cascade. In particular, so long as either information sharing does not occur "too early" or private signals are sufficiently informative, then a player is better off as the common player (and observing the decision of $k-1$ member of each group) than she is by being in *any* position in a basic cascade (and observing the decisions of *any* number of the prior players in her group). In other words, the information revealed from observing the first $k-1$ decisions in each group is more valuable than the information revealed by observing any number of prior decisions in a basic cascade.

Proposition 5. *The payoff of the common player in an interacting cascade exceeds the payoff of every player in a basic cascade, i.e.,* $w(k,k) > w^B(i)$ *for every* $i \in \{1, \ldots, \infty\}$, *if either* $k \geq 5$ *or* $k = 3$ *and* $p \geq \frac{1}{6}\sqrt{3} + \frac{1}{2}$. *If* $k = 3$ *and* $p < \frac{1}{6}\sqrt{3} + \frac{1}{2}$, *then there is an* i' *such that* $w(k,k) < w^B(i)$ *for* $i \geq i'$.

Since payoffs are higher for players moving later in the cascade, i.e., since $w(i,k)$ is increasing in i, we have for $i > k$ that

$$w(i,k) \geq w(k,k) > w^B(i).$$

Since $w(i,k) = w^B(i)$ for $i < k$, the next corollary is immediate.

Corollary 2. If *either* $k \geq 5$ *or* $k = 3$ *and* $p \geq \frac{1}{6}\sqrt{3} + \frac{1}{2}$ *then the payoff of every player in an interacting cascade is higher than the payoff of her counterpart in a basic cascade, i.e.,* $w(i,k) \geq w^B(i)$ *for all* $i \in \{1, \ldots, N\}$.

In other words, under the corollary's assumptions, all players are better off and more likely to make the correct decisions with information spillovers than without.[7]

The Timing of Information Spillovers

If information is shared between groups, *when* should it be shared? There is a trade-off between the number of players who benefit from information sharing and the quality of the information shared. If the common player moves early, then more players subsequently enjoy the benefits of information spillovers. However, if the common player moves later, she and her successors are better informed and more likely to make the correct decision.

We consider positioning the common player in order to maximize total surplus. Total

[7] Analogous results can be shown for alternative measures of well-being – e.g., the probability that a player is in a "correct" cascade (i.e., a cascade on a in state $\mathcal{H}$ and on r in state $\mathcal{L}$) is higher in an interacting cascade than in a basic cascade under the hypotheses like those of Corollary 2.

surplus in an interacting cascade is

$$W(N,p,k) = w(k,k) + \sum_{i \in \{1,\ldots,N\} \setminus \{k\}} 2w(i,k),$$

since there is a single common player, two identical players in the i-th position of each group, and the equilibrium is symmetric. It is never optimal to place the common player first since then she aggregates no information and each player's payoff is the same as her position's payoff in a basic cascade. Thus, our objective is to choose k, where $1 < k < N$ and k is odd, to maximize $W(N,p,k)$. Let $k^*(N,p)$ denote the solution(s).

A useful way to proceed is to think about the effect on payoffs of moving the common player from position k (> 3) to position $k-2$. We do this by picking up the common player, shifting the players occupying positions $k-2$ and $k-1$ one position to the right to fill in the gap, and then inserting the common player into the now empty $k-2$ position. The top panel of Figure 3 depicts the original game and the bottom depicts the new game. The figure shows, for instance, that player $k-2^A$ moves from position $k-2$ to position $k-1$.

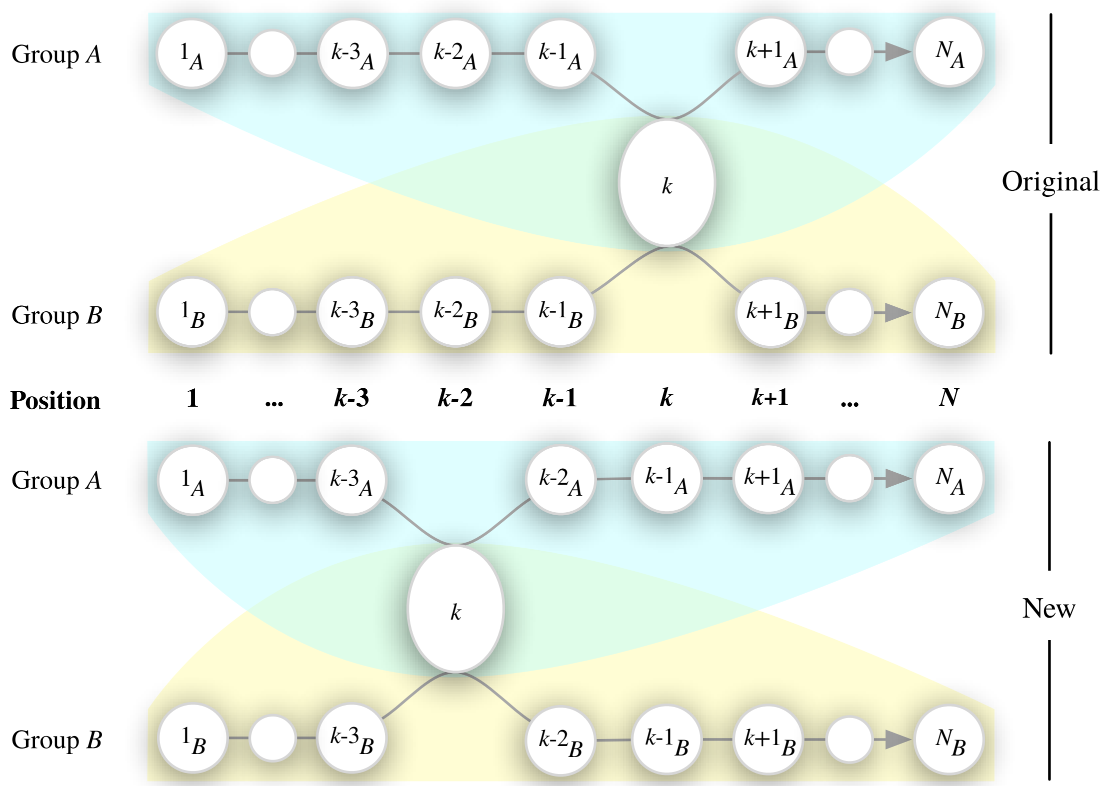

Figure 3: Moving the Common Player

We consider the effect on total surplus of this move. The payoffs of players 1 through

$k-3$ are the same in both games since they do not change position and their payoffs do not depend on the common player's position (see Proposition 4). However, the payoffs of the remaining players change since they depend on the common player's position.

The payoffs of players $k-2$ and $k-1$ (in both groups) increase as they have better information about the state. To see this for player $k-2^g$, recall that in the original game her payoff is $w(k-2,k)$, while in the new game her payoff is $w(k-1,k-2)$. By Proposition 4, we have

$$\begin{aligned} w(k-2,k) &= \frac{2p-1}{2}\Lambda(k-1) \\ &< \frac{2p-1}{2}\Lambda(2k-4) + (2p-1)[\frac{\lambda}{2}\Lambda(k-3)]^2[1+\frac{\lambda}{2}\Lambda(1)] \\ &= w(k-1,k-2), \end{aligned}$$

where the strict inequality holds since Λ is increasing and $k > 3$ implies $2k-4 > k-1$. The common player's payoff decreases as she observes fewer decisions and, thus, less information. The effect on the payoffs of the remaining players is ambiguous since, for $i > k$, the sign of $w(i,k) - w(i,k-2)$ depends on i.[8] For instance, if $p = 0.6$, then player 25^g's surplus increases by 0.001 (from 0.250 to 0.251) when the common player is moved from position 21 to position 19, while player 29^g's surplus decreases by 0.00003 (from 0.25300 to 0.25297). One can show $W(29, .6, 21) = 11.184$ and $W(29, .6, 19) = 11.428$, and hence the effect on total surplus is positive when $N = 29$. The effect on total surplus, however, can be made negative if N is made sufficiently large.

Proposition 6 is our third main result – it shows that, for p sufficiently close to one, total surplus increases when the common player moves from position k to $k-2$ (as in Figure 3). In other words, when the signal accuracy is high, then the benefit of realizing information spillovers earlier exceeds the cost of reduced information aggregation.

Proposition 6. *For each odd $k > 3$, there exists a $p_k < 1$ such that moving the common player from position k to $k-2$ increases total surplus when $p \in [p_k, 1)$, i.e., $W(N,p,k-2) > W(N,p,k)$ for all $p \in [p_k, 1)$.*

It follows immediately that $k^*(N,p) = 3$ for p sufficiently close to 1. In other words, when the signal accuracy is high, then the gains to aggregating the information of more

[8] The intuition is two-fold. First, moving the common player earlier causes her to aggregate less information; all else equal, this reduces the payoffs of later players. Second, each player moving after the common player benefits from the insertion of players between herself and the common player (since they allow for additional information aggregation when the common player breaks the group's cascade). The size of this benefit, however, depends on the player's distance from the common player – it is substantial if she is close and is insignificant if she is far. The sign of $w(i,k) - w(i,k-2)$ is therefore positive for i near k and negative otherwise.

than the first two players in each group are more than offset by the benefits from realizing information spillovers immediately.

Corollary 3. *There is a* $p_N < 1$ *such that* $k^*(N, p) = 3$ *for all* $p \in [p_N, 1)$.

Table 1 illustrates how $W(N, p, k)$ depends on p and k when $N = 15$ (i.e., there are 14 players in each group and one common player).

p	$k = 3$	$k = 5$	$k = 7$	$k = 9$	$k = 11$	$k = 13$
0.55	2.583	2.900	**2.928**	2.851	2.737	2.617
0.65	7.342	8.033	**8.054**	7.865	7.611	7.351
0.75	11.016	**11.566**	11.511	11.321	11.102	10.883
0.85	13.275	**13.440**	13.371	13.278	13.183	13.089
0.95	**14.273**	14.266	14.255	14.245	14.234	14.223

Bold indicates a maximum

Table 1: $W(15, p, k)$

If $p = 0.55$, for example, then total surplus is maximized with the common player in position 7. The table illustrates that, as signals become more informative, the optimal position of the common player moves earlier.

There is a "dual" result to Corollary 3: for each integer $n \geq 3$, there is a $p' > \frac{1}{2}$ and a N' such that $\min k^*(N, p) > n$ for all $p \in (\frac{1}{2}, p']$ and $N > N'$. That is, it is surplus maximizing to delay information aggregation via the common player when p is close to $1/2$ and the number of players is large. This result is not particularly surprising in light of our previous discussion and Table 3, so we omit the proof.

4 Discussion and Conclusion

We have shown that when groups share a player in common then (i) a cascade may spillover from one group to another via the common player, (ii) a cascade in a group will be broken if it disagrees with the decision of the common player, and (iii) a cascade can be triggered by the decision of the common player when no prior cascade existed. We have also shown that information spillovers via a common player are usually, but not always, welfare enhancing and that total surplus is maximized when the common player moves early (late) when the signal accuracy is high (low).

While our simple network lacks the complexity of many real-world networks, it captures several key characteristics – e.g., strong within-group ties and limited between-group ties – and can easily be embedded in a larger network – allowing our results to be recast as local results. Our model's strength is its simplicity, which allows us to completely characterize equilibrium behavior and examine information spillovers.

We conclude with a discussion of the robustness of our results. When the common player is in an even position, the structure of equilibrium is slightly more complex as an odd number of players move (in each group) prior to the common player. Thus, when the common player observes that a group is not in a cascade, she no longer finds their history uninformative due to an equal number of a and r decisions; rather, the history contains an "extra" a or r decision from the $k-1$-**st** player and so it is informative. Thus the common player's decision depends on the decisions of $k-1^A$ and $k-1^B$, e.g., if group A is in a cascade on a and group B is not in a cascade, she chooses r if $d^B_{k-1} = r$ and $x_k = L$ and chooses a otherwise. In the former case, which occurs with probability $P(L|s)^2$ in state s, the common player's decision breaks the cascade on a in group A, while in the later case, which occurs with probability $1 - P(L|s)^2$ in state s, the cascade in group A spills over to group B. Under assumptions analogous to those in Section 3, one can show that (i) cascades continue to spillover, break, and be triggered, (ii) the common player's payoff still exceeds those of the basic cascade players, and (iii) social welfare still increases when moving the common player to an earlier position; details are available upon request.

Our assumption that the each player's signal has the same precision is not essential. If instead the signal precisions of the members of groups A and B (excluding the common player) are p^A and p^B, respectively, and that of the common player is p, then Propositions 1 through 3 go through without modification provided that p^A, p^B, and p are close. Since the equilibrium is invariant to small changes in the signal precisions, the equilibrium payoffs (Proposition 4) are continuous functions of p^A, p^B, and p in the neighborhood of $p^A = p^B = p$. Thus, as in Proposition 5 and 6, (i) either $k \geq 5$ or $p > \frac{1}{6}\sqrt{3} + \frac{1}{2}$ are sufficient for the common player's payoff to exceed the payoff of every player in a basic cascade and (ii) moving the common player from k to $k-2$ is welfare improving when p is large, provided p^A and p^B are in a neighborhood of p. However, when p^A, p^B, and p are not close, the results can differ. For instance, when $N = 4$, $k = 3$, $p^A = 4/5$, $p^B = 2/3$, and $p = 4/5$, it is easily seen that (i) cascades spillover from group A to group B (when B is not in a cascade) but not the reverse and (surprisingly!) that (ii) the common player is still able to break cascades in A.[9]

Propositions 1 through 3 also go through without modification when the common player is randomly positioned in each group, so long as each player i^g observes the position of the common player in her group.[10] (It does not matter whether i^g observes the position of the

[9] The details for all of the examples discussed are available upon request.

[10] While it is easily seen that cascades may spill over, may be broken, and may be triggered by the common player when her position is unobserved by any other players (before play begins), the specifics depend on her realized position; details are available upon request.

common player in the other group.) However, the expression for payoffs in Proposition 4 relies on the common player occupying the same position in both groups. Thus, one would need to develop new expressions for payoffs and new sufficient conditions for the payoff of the common player to exceed those of all the other players.

Analogues of Propositions 1 to 3 obtain when the states are not equally likely. Specifically, let $q > \frac{1}{2}$ be the probability of an $\mathcal{H}$ state and $1 - q$ be the probability of an $\mathcal{L}$ state. Then, the analogues obtain when (i) $q < p$ and (ii) $q < \frac{\alpha + p\beta}{2\alpha + \beta}$, where $\alpha_k = (\frac{\lambda}{2})^{\frac{k-1}{2}}$ and $\beta_k = \frac{1-\alpha}{1-\frac{\lambda}{2}}$. The first condition ensures that a player follows her signal in the absence of all other information. The second condition is a joint requirement (q, p, k) that is met, for instance, when q is "sufficiently close" to $\frac{1}{2}$ given p and k; it ensures that players moving after the common player are not so biased in favor of an $\mathcal{H}$ state that their behavior is invariant to the common player's decision. When these conditions hold, cascades spillover and are triggered as in Proposition 3. However, when $q > \frac{1}{2}$, cascades break asymmetrically. To illustrate, suppose A and B are in cascades on a and r, respectively. Then, the common player follows her signal. If she chooses a, then the cascade in A continues and the cascade in B ends. If, however, she chooses r, then the cascade in B continues and the cascade in A is *replaced* with a cascade on r (i.e., $k + 1^A$ and all subsequent players in A are in a cascade on r). The key insight is that cascades on a are less informative than cascades on r because of the state asymmetry. The details of this extension are provided in the Online Appendix.

Analogues of Propositions 1 to 3 also hold when there are (i) multiple common players or (ii) multiple groups. To illustrate (i), suppose that two groups, each consisting of six players, share common players in the third and fifth positions. Then, it is readily verified that cascades before the first common player spill over exactly as in Proposition 3. Differences emerge, however, with respect to how cascades are triggered and broken. If neither group is in a cascade before the first common player, then both common players follow their signals and trigger a cascade when their decisions agree; if their decisions disagree, some later players follow their own signals. Further, if both groups are in opposing cascades before the first common player, then *both* cascades may break: when the common players follow their signals and these signals disagree, players 6^A and 6^B each see a decision by a common player that contradicts the early cascade in her group and follows her own signal.

To illustrate (ii), suppose that three groups, each consisting of four players, share a common player in the third position. Then (i) players 1^g and 2^g behave as in Proposition 1, (ii) the common player is in a cascade when one group is in a cascade and the other two are not or when two groups are in the same cascade, but she otherwise follows her own signal, and (iii) player 4^g is in a cascade on the common player's decision (regardless of the decisions of 1^g and 2^g). Thus, interaction is similar to Proposition 3 – e.g., (a) if no group is in a cascade, then one starts on the common player's decision and (b) if one group is in a cascade and the others are not, then the cascade spills over. There are, however, differences – e.g., (c) if two groups are in the same cascade, then this cascade spills over to the third group regardless of whether it was in a cascade.

5 Appendix: Equilibrium Characterization & Proofs

We begin with a few definitions, present three lemmas which completely characterize equilibrium play, and the prove our results. Before we begin, we need some notation. A strategy for player i^g is a function $\sigma_i^g : \{a, r\}^{i-1} \times \{H, L\} \to \{a, r\}$ that maps the profile of decisions of prior players and her own private signal into a decision, and a strategy for player k is a function $\sigma_k : \{a, r\}^{2(k-1)} \times \{H, L\} \to \{a, r\}$. (Our characterization omits beliefs since these can be easily recovered with Bayes' Rule.)

A history $\bar{d}_i = (d_1, \ldots, d_i)$ is **balanced** if, for every odd integer $j < i$, we have that $d_j \neq d_{j+1}$, and is **unbalanced** otherwise. A balanced history is one where the cumulative number of a (r) decisions does not exceed the cumulative number of r (a) decisions by more than one as of any player $1, \ldots, i$. The empty history and any singleton history are trivially balanced. A history $\bar{d}_i = (d_1, \ldots, d_i)$ is **unbalanced on *a*** if at some point in the profile it switches from a balanced profile to a profile of only a, i.e., if there is an odd $j < i$ such that (i) $(d_1, \ldots, d_j)$ is balanced and (ii) $d_j = d_{j+1} = \cdots = d_i = a$. Let D_i^a be the set of i-length histories that are unbalanced on a. Likewise, a history $\bar{d}_i = (d_1, \ldots, d_i)$ is **unbalanced on *r*** if there is an odd $j < i$ such that (i) $(d_1, \ldots, d_j)$ is balanced and (ii) $d_j = d_{j+1} = \cdots = d_i = r$. Let D_i^r be the set of i-length histories that are unbalanced on r.

Lemma A1. Equilibrium play for predecessors of the common player.
Let $i < k$ and $g \in \{A, B\}$. In equilibrium, $\bar{d}_{i-1}^g$ belongs to a row in Table A1(a) and player

i^g's equilibrium strategy $\sigma_i^{g*}(\bar{d}_{i-1}^g, x_i^g)$ is given by the last two columns.

$\bar{d}_{i-1}^g$	$\sigma_i^{g*}(\bar{d}_{i-1}^g, x_i^g)$ H	L
D_{i-1}^a	a	a
D_{i-1}^b	a	r
D_{i-1}^r	r	r

(a) Player $i^g < k$

$\bar{d}_{k-1}^A$	$\bar{d}_{k-1}^B$	$\sigma_k^*(\bar{d}_{k-1}^A, \bar{d}_{k-1}^B, x_k)$ H	L
D_{k-1}^a	D_{k-1}^a	a	a
	D_{k-1}^b	a	a
	D_{k-1}^r	a	r
D_{k-1}^b	D_{k-1}^a	a	a
	D_{k-1}^b	a	r
	D_{k-1}^r	r	r
D_{k-1}^r	D_{k-1}^a	a	r
	D_{k-1}^b	r	r
	D_{k-1}^r	r	r

(b) Player k

Table A1: Equilibrium Strategies of Players 1 through k

Proof. The proof is due to Bikhchandani et al. (1992). □

For instance, if player i observes a history that is unbalanced on a then she chooses a, ignoring her own signal (i.e., if $\bar{d}_i^g \in D_{i-1}^a$, then Table A1(a) shows that $\sigma_i^{g*}(\bar{d}_{i-1}^g, H) = \sigma_i^{g*}(\bar{d}_{i-1}^g, L) = a$). If player i observe a balanced history, then she follows her own signal.

Lemma A2. Equilibrium play for the Common Player.
In equilibrium, $(\bar{d}_{k-1}^A, \bar{d}_{k-1}^B)$ belongs to a row in Table A1(b) and the common player's equilibrium strategy $\sigma_k^*(\bar{d}_{k-1}^A, \bar{d}_{k-1}^B, x_k)$ is given by the last two columns.

Proof. The proof is computational. Successive application of Lemma A1 lets us enumerate the set of equilibrium histories and compute the equilibrium probability of each history. We then employ Bayes Rule to compute the common player's belief and write the table. For details, see the Online Appendix. □

The next proposition identifies the behavior of players moving after the common player.

Lemma A3. Equilibrium After the Common Player.
Let $g \in \{A, B\}$.
LA3.1 : (Player $k+1^g$.) In equilibrium, $\bar{d}_k^g = (\bar{d}_{k-1}^g, d_k)$ belongs to a row in Table A2(a) and player $k+1^g$'s equilibrium strategy $\sigma_{k+1}^{g*}(\bar{d}_k^g, x_{k+1}^g)$ is given by the last two columns.
LA3.2 : (Player $k+2^g$.) In equilibrium, $\bar{d}_{k+1}^g = (\bar{d}_{k-1}^g, d_k, d_{k+1}^g)$ belongs to a row in Table A2(b) and player $k+2^g$'s equilibrium strategy $\sigma_{k+2}^{g*}(\bar{d}_{k+1}^g, x_{k+2}^g)$ is given by the last two columns.

LA3.3 : (Subsequent players.) Let $i > k+2$. In equilibrium, $\bar{d}^g_{i-1} = (\bar{d}^g_{k-1}, d_k, d^g_{k+1}, d^g_{k+2}, \ldots, d^g_{i-1})$ belongs to a row in Table A2(c) and player i^g's equilibrium strategy $\sigma^{g*}_i(\bar{d}^g_{i-1}, x^g_i)$ is given by the last two columns.

		$\sigma^{g*}_{k+1}(\bar{d}^g_k, x^g_{k+1})$	
$\bar{d}^g_{k-1}$	d_k	H	L
D^a_{k-1}	a	a	a
	r	a	r
D^b_{k-1}	a	a	a
	r	r	r
D^r_{k-1}	a	a	r
	r	r	r

(a) Player $k + 1^g$

			$\sigma^{g*}_{k+2}(\bar{d}^g_{k+1}, x^g_{k+2})$	
$\bar{d}^g_{k-1}$	d_k	d^g_{k+1}	H	L
	a	a	a	a
D^a_{k-1}	r	a	a	r
	r	r	r	r
D^b_{k-1}	a	a	a	a
	r	r	r	r
	a	a	a	a
D^r_{k-1}	a	r	a	r
	r	r	r	r

(b) Player $k + 2^g$

				$\sigma^{g*}_i(\bar{d}^g_{i-1}, x^g_i)$	
$\bar{d}^g_{k-1}$	d_k	d^g_{k+1}	$(d^g_{k+2}, \ldots, d^g_{i-1})$	H	L
	a	a	$(a, \ldots, a)$	a	a
			D^a_{i-k-2}	a	a
D^a_{k-1}	r	a	D^b_{i-k-2}	a	r
			D^r_{i-k-2}	r	r
	r	r	$(r, \ldots, r)$	r	r
D^b_{k-1}	a	a	$(a, \ldots, a)$	a	a
	r	r	$(r, \ldots, r)$	r	r
	a	a	$(a, \ldots, a)$	a	a
			D^a_{i-k-2}	a	a
D^r_{k-1}	a	r	D^b_{i-k-2}	a	r
			D^r_{i-k-2}	r	r
	r	r	$(r, \ldots, r)$	r	r

(c) Player $i^g > k + 2^g$

Table A2: Equilibrium Strategies of Players $k + 1$ through N

For instance, if $\bar{d}^g_{k-1} \in D^a_{k-1}$ and $d_k = a$, then the top row of Table A2(a) shows that player $k + 1^g$ is in a cascade on a.

Proof. The proof is a computational exercise that mirrors the Proof of Lemma A2. The details are in the Online Appendix. □

Proof of Proposition 1. Lemma A1 shows that, in equilibrium, player i observes a history $\bar{d}_{i-1}$ in either D^a_{i-1}, D^b_{i-1}, or D^r_{i-1}. For histories where the number of a decisions exceeds the number of r decisions by two or more, we have $\bar{d}_{i-1} \in D^a_{i-1}$ and player i chooses a by Table A1. Likewise, for histories $\bar{d}_{i-1} \in D^r_{i-1}$ the number of r decisions exceeds the number of a decisions by two or more, and she chooses r. Otherwise, player i observes a history in D^b_{i-1} in which case she follows her own signal. □

Proof of Proposition 2. Follows directly from Lemma A2 since a cascade on a (r) occurs when a group's history is in D^a_{k-1} (D^r_{k-1}) by Lemma A1. □

Proof of Proposition 3. Follows from Lemmas A2 and A3. We illustrate with P3.1: Since both groups are in cascades on a, we have that $\bar{d}^A_{k-1}$ and $\bar{d}^B_{k-1}$ are in D^a_{k-1}. Thus, Lemma A2 gives that the common player chooses a. Hence, successive application of Lemma A3 gives that every subsequent player in group g chooses a. The remaining cases are analogous. □

Proof of Proposition 4. The proof is a computation exercise and omitted. The details are in the Online Appendix. □

Proof of Proposition 5. In a basic cascade, the players' asymptotic payoff is

$$\lim_{i\to\infty} w^B(i) = \lim_{i\to\infty} \frac{2p-1}{2}\Lambda(i) = \frac{2p-1}{2(1-\lambda)}.$$

In an interacting cascade, the payoff of the common player is

$$w(k,k) = \frac{2p-1}{2(1-\lambda)}\left[1-\lambda^k + \frac{1}{2(1-\lambda)}[\lambda(1-\lambda^{\frac{k-1}{2}})]^2\right].$$

Hence, $w(k,k) \geq \lim_{i\to\infty} w^B(i)$ if and only if

$$1-\lambda^k + \frac{1}{2(1-\lambda)}[\lambda(1-\lambda^{\frac{k-1}{2}})]^2 \geq 1,$$

equivalently,

$$(1-\lambda)(1-3\lambda^{k-2}) + \lambda^{-2}(\lambda^{\frac{3}{2}} - \lambda^{\frac{k}{2}})^2 \geq 0.$$

Since $p \in (1/2, 1)$, then $\lambda = 2p(1-p) \in (0, 1/2)$. If $k \geq 5$, then $1-3\lambda^{k-2} > 0$ and $\lambda^{\frac{3}{2}} - \lambda^{\frac{k}{2}} > 0$, and hence $w(k,k) > \lim_{i\to\infty} w^B(i)$. While if $k = 3$, then $\lambda^{\frac{3}{2}} - \lambda^{\frac{k}{2}} = 0$ and $1 - 3\lambda \geq 0$ if $\lambda \leq 1/3$ (i.e., $p \geq \frac{1}{6}\sqrt{3} + \frac{1}{2}$), and hence $w(k,k) \geq \lim_{i\to\infty} w^B(i)$. Since $w^B(j) < \lim_{i\to\infty} w^B(i)$ for each j, then $w(k,k) > w^B(i)$ for all i if either $k \geq 5$ or $k = 3$ and $p \geq \frac{1}{6}\sqrt{3} + \frac{1}{2}$.

If $k = 3$ and $p < \frac{1}{6}\sqrt{3} + \frac{1}{2}$ then $w(k,k) < \lim_{i\to\infty} w^B(i)$, so there is some i such that $w(k,k) < w^B(i)$. □

Proof of Proposition 6. We prove the result by establishing that the gain from moving the common player exceeds the loss when p is large. Since players in positions 1, ..., $k-3$ in both groups are unaffected by the move, we only need to show that the (i) the minimum gain for players $k-2^A$, $k-1^A$, $k-2^B$, and $k-1^B$ exceeds the (ii) maximum loss for the common player and players $k+1^A$, $k+1^B$, ..., N^A, and N^B. With this in mind, we proceed by constructing a (i') lower bound on the gain and (ii') an upper bound on the loss. We then establish that (i') is strictly greater than (ii') when p is sufficiently close to 1. The desired result follows.

First, we compute a lower bound on the gain for players $k-2^A$, $k-1^A$, $k-2^B$, and $k-1^B$. Table A3 computes the gain to each of these players.

Player	Gain
$k-2$	$w(k-1,k-2)-w(k-2,k)=\frac{2p-1}{2}\lambda^{\frac{k-1}{2}}\Lambda(k-3)+(2p-1)(\Lambda(k-3)\frac{\lambda}{2})^2$
$k-1$	$w(k,k-2)-w(k-1,k)=\frac{2p-1}{2}\lambda^{\frac{k-1}{2}}\Lambda(k-3)+(2p-1)(\Lambda(k-3)\frac{\lambda}{2})^2(1+\frac{\lambda}{2})$

Table A3: Gain to Players $k-2$ and $k-1$

It is clear from the table that the total gain to all four players is at least $G(p)=2(2p-1)\lambda^{\frac{k-1}{2}}\Lambda(k-3)$.

Second, we compute an upper bound on the loss for the common player and subsequent players in both groups. Table A4 computes the loss to each of these players.

Player	Loss
Common Player	$w(k,k)-w(k-2,k-2)=$ $\frac{2p-1}{2}\lambda^{k-2}\Lambda(4)+(2p-1)(\frac{\lambda}{2})^2(\Lambda(k-1)^2-\Lambda(k-3)^2)$
$k+1$	$w(k+1,k)-w(k+1,k-2)=$ $\frac{2p-1}{2}\lambda^{k-2}\Lambda(4)+(2p-1)(\frac{\lambda}{2})^2(\Lambda(k-1)^2-\Lambda(k-3)^2(1+\frac{\lambda}{2}))$
$i\geq k+2$ and i odd	$w(i,k)-w(i,k-2)=\frac{2p-1}{2}\lambda^{k-2}\Lambda(4)+(2p-1)(\frac{\lambda}{2})^2[$ $\Lambda(k-1)^2(1+\frac{\lambda}{2}\Lambda(i-k))-\Lambda(k-3)^2(1+\frac{\lambda}{2}\Lambda(i-k+2))]$
$i\geq k+2$ and i even	$w(i,k)-w(i,k-2)=w(i-1,k)-w(i-1,k-2)$

Table A4: Loss to Common Player and Subsequent Players

A bit of algebra shows that every row of Table A4 is less than or equal to

$$T(p)=\frac{2p-1}{2}\lambda^{k-2}\Lambda(4)+2(2p-1)(\frac{\lambda}{2})^2(\Lambda(k-1)^2-\Lambda(k-3)^2).$$

Consequently, the loss to the common player and all subsequent players is at most $L(p)=$

$2(N-k+\frac{1}{2})T(p)$.

We now establish that $G(p) \geq L(p)$ for p sufficiently close to 1, this then implies that the move of the common player is surplus improving. To do this, consider the related inequality

$$\underbrace{2(1-\lambda^{\frac{k-3}{2}})}_{\mathrm{LHS}(p)} \geq \underbrace{(N-k+\frac{1}{2})\lambda^{\frac{k-3}{2}}(1-\lambda^2)}_{\mathrm{RHS}_1(p)} + \underbrace{(N-k+\frac{1}{2})(1-\lambda)\lambda^{\frac{5}{2}}(\frac{\Lambda(k-1)^2}{\lambda^{k/2}} - \frac{\Lambda(k-3)^2}{\lambda^{k/2}})}_{\mathrm{RHS}_2(p)}. \quad (1)$$

Observe that $G(p) = \frac{2p-1}{1-\lambda}\lambda^{\frac{k-1}{2}}\mathrm{LHS}(p)$ and that $L(p) = \frac{2p-1}{1-\lambda}\lambda^{\frac{k-1}{2}}(\mathrm{RHS}_1(p) + \mathrm{RHS}_2(p))$. Note that (i) $\mathrm{LHS}(1) = 2$, (ii) $\mathrm{RHS}_1(1) = 0$, and (iii) $\mathrm{RHS}_2(1) = 0$. The first and second equalities are obvious. The third equality follows from the fact that $(1-\lambda)\lambda^{\frac{5}{2}}(\frac{\Lambda(k-1)^2}{\lambda^{k/2}} - \frac{\Lambda(k-3)^2}{\lambda^{k/2}}) = 2\lambda - \lambda^{\frac{k-1}{2}}(1+\lambda)$, which equals 0 when $p = 1$ since $k > 1$. It follows that (1) is true with strict inequality when $p = 1$. Since both sides of (1) are continuous in p, there is a p_k, with $\frac{1}{2} < p_k < 1$, such that (1) is true for all $p \geq p_k$. At every $p \in [p_k, 1)$, we have that $\frac{2p-1}{1-\lambda}\lambda^{\frac{k-1}{2}} > 0$. Multiplying both sides of (1) by $\frac{2p-1}{1-\lambda}\lambda^{\frac{k-1}{2}}$ shows that $G(p) \geq L(p)$. $\square$

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