Effects of One-Way Spillovers on Market Shares, Industry Price, Welfare, and R & D Cooperation

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With one-way spillovers, the standard symmetric two-period R & D model leads to an asymmetric equilibrium only, with endogeneous innovator and imitator roles. We show how R & D decisions and measures of firm heterogeneity—market shares, R & D shares, and profits—depend on spillovers and on R & D costs. While a joint lab always improves on consumer welfare, it yields higher profits, cost reductions, and social welfare only under extra assumptions, beyond those required with multidirectional spillovers. Finally, the novel issue of optimal R & D cartels is addressed. We show an optimal R & D cartel may seek to minimize R & D spillovers between its members.

1. Introduction

The starting point for the present paper is the standard two-period model of process-R & D–product-market competition for an *ex ante* symmetric duopoly under imperfect appropriability of R & D. Rather than assuming that each firm learns something from the rival firm in the course of conducting R & D, we depart from existing models in this area by postulating a stochastic directed spillover process. In this vision, know-how may only flow from the more R & D-intensive firm

The authors thank Claude d'Aspremont, David Audretsch, Jim Jin, Kai-Uwe Kühn, Stanley Reynolds, Lars-Hendrik Röller, Bernard Sinclair-Desgagné, Yair Tauman, Peter Zemsky, and two anonymous referees and a coeditor of this journal, as well as seminar participants at C.O.R.E., INSEAD, the University of Arizona, the Free and Humboldt Universities, Berlin, the W.Z.B./C.E.P.R. Conference on Industrial Policy, and the 1st Conference on Industrial Organization at Carlos III—Madrid, for their comments and suggestions.

Journal of Economics & Management Strategy, Volume 8, Number 2, Summer 1999, 223–249

to the rival, but never in the opposite direction. Furthermore, these flows of R & D know-how are probabilistic in a binomial sense: With probability β (defined as the spillover parameter), full spillover occurs, and with probability $1 - \beta$, no spillover occurs. In the former case, the two firms end up with the same unit cost, while in the latter case, each firm ends up exactly with its autonomous cost reduction. In view of the central role of this feature of our model, a detailed justification and interpretation is presented later.

This work is motivated by issues originating from two independent areas of applied microeconomics, one dealing with intra-industry heterogeneity and the other with strategic R & D and research joint ventures (RJVs). In order to relate this paper more precisely to these two areas, we begin with a brief literature summary for each area, followed by an outline of the contribution of this paper.

Variability across firm characteristics within a given industry is a ubiquitous phenomenon. Firms tend to differ in several ways, including product variety, advertising strategy, corporate culture, organizational forms, incentive/compensation schemes, R & D strategy, etc. More obviously, they often differ in their size, market conduct, and overall performance. Economists have long sought to reconcile this observed heterogeneity with conventional economic wisdom. A thorough yet concise overview of the various approaches is given by Röller and Sinclair-Desgagné (1996).

Our contribution to this area may be described as follows. Although our model postulates *ex ante* identical firms, the only pure-strategy subgame-perfect equilibria are asymmetric, thereby yielding endogenous roles of R & D innovator (the more R & D-intensive firm) and R & D imitator.[1] This heterogeneity would naturally translate into the firms pursuing different R & D strategies, choosing different R & D compositions, and running labs of different types and sizes.[2] Furthermore, in cases where full spillover is not realized (*ex post*), the firms will also end up with different unit costs, and hence different market shares. According to this perspective, the mere knowledge that R & D leakages flow (in a stochastic sense) only from the more R & D-intensive firm to the rival leads firms to (endogenously) settle for innovator and imitator roles, thereby trading off

1. It can be shown that there is also a symmetric mixed-strategy equilibrium. In such an equilibrium, the firms also end up (endogenously) different with probability one.

2. A number of recent studies have dealt with endogenous asymmetry in various contexts, including Hermalin (1994), Boyer-Moreaux (1997), Choi (1993), and Katz and Shapiro (1987), the last two dealing with setups related to our own.

profits in the product market and R & D costs in complementary ways.

We also explore the nature of intra-industry heterogeneity in detail, examining how each firm's R & D decision, market share, and R & D expenditure as a share of industry expenditure depend on the spillover rate β and the cost of R & D. In particular, we find the innovator's and imitator's R & D decisions are sometimes increasing in β, although this behavior can be ruled out if R & D costs are sufficiently convex or demand is high relative to the initial unit cost.

Firms also differ in realized profit. We identify the conditions under which the innovator has a higher expected profit than its rival. We also show that at $\beta = 0$ (i.e., R & D is perfectly appropriable), each firm's profit is increasing in β. Hence, *ex ante* both firms prefer some degree of imperfect appropriability of R & D to none. In contrast, when $\beta = 1$ (i.e., R & D is a pure public good), as β falls, the innovator's profit rises, while the imitator's profit falls unless R & D costs are low.

We now provide a brief overview of the literature on RJVs. The central aim of this literature is to provide a performance comparison between various R & D cooperation scenarios, ranging from full cooperation to pure (strategic) competition, among firms which remain competitors in the product market. See Katz (1986); d'Aspremont and Jacquemin (1988, 1990), henceforth AJ; and Kamien et al. (1992), henceforth KMZ.[3] The main result is that the cartelized RJV, which may be viewed as a situation where firms run one joint R & D lab at equal cost to each, yields the best performance among all scenarios considered, in terms of R & D propensity, consumer surplus, and producer surplus.[4] Several studies have built on the results of AJ and KMZ to address related questions, including Suzumura (1992), DeBondt et al. (1992), and Salant and Shaffer (1998).[5]

3. Earlier studies, such as Brander and Spencer (1983), Dagupta and Stiglitz (1980), and Spence (1984), do not consider R & D cooperation.

4. This result is valid for Cournot and Bertrand competition with differentiated products, but in the latter case only for cross-slopes coefficients less than or equal to $\frac{2}{3}$ under linear demand (see KMZ, p. 1305). It is easily seen that the result would not hold for Bertrand competition with homogeneous products.

5. More recently, another set of papers only partially confirms some of the accepted results in the literature: Isaac and Reynolds (1988), Reynolds and Isaac (1992), and Stenbacka and Tombak (1998) consider models with stochastic R & D processes, and Amir (1995) deals with the standard model, but with R & D returns that are not strongly decreasing. With their respective modifications, the latter two studies report a reduced scope of validity for the superiority of the cartelized RJV. See also Amir and Wooders (1998).

In these studies, spillovers were always treated as a deterministic multidirectional process: a fixed proportion (given by the spillover parameter) of every firm's R & D effort or benefit flows freely to (all) the rival(s). As argued by KMZ, the underlying R & D process in these studies is implicitly assumed to be a ''multidimensional heuristic rather than a one-dimensional algorithmic process.'' Thus, it necessarily involves trial and error on the part of the firms, which follow different sets of research paths and/or approaches. By contrast, the R & D process associated with the one-way (or undirectional) spillover structure here is best approximated by a one-dimensional process, i.e., there is a single research path firms might follow in order to reduce unit costs. In case of a multipath R & D process, our spillover structure suggests the presence of a more or less natural order on the various steps to be performed. In either setting it is natural to postulate that the only spillover potential is from the firm with higher R & D activity to the laggard(s).

Our contribution to this second area is that while the endogenous asymmetry can reverse some of the established conclusions on the superiority of the joint lab, these conclusions can be restored via a strengthening of the basic assumptions of the model. Specifically, contrary to the established conclusions, we find that under R & D competition a firm (the innovator) sometimes conducts more R & D than the joint lab, and the sum of firm profits is sometimes higher than under the joint lab. To restore the established conclusions requires new assumptions that can be interpreted as saying that R & D costs and/or demand must be sufficiently high. We show that consumer surplus is always higher under the joint lab. Therefore the same assumptions ensuring that the sum of profits is higher under the joint lab also ensure that social welfare, defined as the sum of consumer surplus and firm profits, is higher under the joint lab.

Previous studies of R & D cartels have either taken the spillover parameter to be beyond the control of the cartel or assumed that cartel members coordinate their R & D efforts and fully share R & D results, i.e., the spillover parameter is one. (KMZ refer to these cases as R & D and RJV cartelization, respectively. They also bring up the issue of optimal R & D cartels.) In Section 4 we characterize the structure of R & D cartels when, in addition to choosing (possibly asymmetric) cost reductions, a cartel also chooses the spillover rate s in $[0, 1]$. We show that the optimal R & D cartel has either no or full spillovers, i.e., either $s^* = 0$ or $s^* = 1$. When the optimal cartel has full spillovers, then one firm conducts the same level of R & D as the joint lab, while the other firm conducts none, and each firm enjoys the same final cost reduction as obtained with a joint lab. (The only difference between the R & D cartel with $s = 1$ and the joint lab is that R & D costs need not be equally shared in the cartel.) When the optimal cartel has no spillovers, then firms are maximally differentiated, one firm undertaking the maximal possible cost reduction, the other firm not reducing its costs at all, and each firm's autonomous cost reduction is its final cost reduction.

The paper is organized as follows. Section 2 introduces one-way spillovers and characterizes equilibrium under R & D competition. Section 3 compares R & D competition and R & D cooperation via a joint lab. Section 4 studies optimal R & D cartels. Some extensions of our model are discussed in Section 5. Concluding remarks are provided in Section 6.

2. NONCOOPERATIVE R & D

2.1. THE MODEL

Consider an industry in which two a priori identical firms, each with unit cost $c > 0$, engage in a two-stage game. At the first stage, firms 1 and 2 conduct process R & D, choosing amounts x_1 and x_2, respectively, $x_1, x_2 \in [0, c]$, by which each reduces its unit cost of production. The cost to firm i of a unit cost reduction of x_i is $\gamma x_i^2/2$. At the second stage the firms are Cournot competitors in an output market.

A crucial feature of the model is the imperfect appropriability of R & D results. Specifically, given autonomous cost reductions of x_i by firm i and x_j by its rival with $x_i \geq x_j$, say, then the firms' effective (or final) cost reductions are X_i and X_j, respectively, where

$$X_i = x_i \quad \text{and} \quad X_j = \begin{cases} x_i & \text{with probability } \beta, \\ x_j & \text{with probability } 1 - \beta. \end{cases} \tag{1}$$

With this formulation of the spillover process, the parameter $\beta \in [0, 1]$ is the probability that the R & D results of the firm choosing the greater autonomous cost reduction spillover to its rival. We provide interpretations in the next subsection.

At the second stage, each firm observes its own and its rival's realized effective cost reduction, and then the firms compete by choosing quantities of a (homogenous) output. The firms face a linear inverse demand function with quantity units normalized so that (with q_i denoting the output of firm i) $P(q_1, q_2) = a - (q_1 + q_2)$.

A (pure) strategy for firm i is a pair (x_i, q_i), where $x_i \in [0, c]$ and $q_i : [0, c]^2 \to \mathbb{R}^+$ is a map from effective cost reductions into outputs. Attention is restricted to subgame-perfect equilibria.

The following assumptions are in effect throughout the paper.

A1: Demand is sufficiently high relative to initial unit cost: $a > 2c$.

A1 ensures that every subgame at the second stage has a unique Nash equilibrium (with both firms in the market). Specifically, in the subgame where firm i's cost is c_i and its rival's cost is c_j, where $c_i, c_j \in [0, c]$, firm i's Cournot output and profit are, respectively, $(a - 2c_i + c_j)/3$ and $(a - 2c_i + c_j)^2/9$.

Firm i's profit in the overall game, denoted by $F(x_i, x_j)$, is its profit at the second stage net of its R & D costs, i.e.,

$$F(x_i, x_j) = \begin{cases} \dfrac{\beta}{9}(a - c + x_i)^2 + \dfrac{1-\beta}{9}(a - c + 2x_i - x_j)^2 - \dfrac{\gamma}{2}x_i^2 \\ \quad \text{if} \quad x_i \geq x_j, \\ \dfrac{\beta}{9}(a - c + x_j)^2 + \dfrac{1-\beta}{9}(a - c + 2x_i - x_j)^2 - \dfrac{\gamma}{2}x_i^2 \\ \quad \text{if} \quad x_i \leq x_j. \end{cases} \tag{2}$$

This expression reflects that when $x_i \geq x_j$, say, then with probability β a spillover occurs, in which case each firm obtains the greater cost reduction x_i and each has a second-stage profit of $(a - c + x_i)^2/9$. With probability $(1 - \beta)$ a spillover does not occur, in which case each firm obtains its own autonomous cost reduction and firm i has a second-stage profit of $(a - c + 2x_i - x_j)^2/9$.

Since the second-stage game has a unique Nash equilibrium, every Nash equilibrium of the game with payoffs (2) induces a subgame-perfect equilibrium of the two-stage game, and vice versa. In view of this one-to-one correspondence, in the following we refer to the Nash equilibria of the game with payoffs (2) rather than the subgame-perfect equilibrium of the two-stage game.

Define the best-response functions in the usual way. For firm i, say, and $x_j \in [0, c]$ let $r_i(x_j) = \arg\max\{F(x_i, x_j): x_i \in [0, c]\}$. Since the game is symmetric, the firm's reaction functions are the same, i.e., $r_i(x) = r_j(x)$, and thus to reduce notation we write $r(x)$ for a firm's best response to an autonomous cost reduction of x by its rival.

A2: R & D costs are sufficiently convex that (i) $9\gamma > 4(a/c)(1 - \beta)$ and (ii) $9\gamma > 8 - 6\beta$.

A2(i) ensures that firm i's best response to an autonomous cost reduction of c by its rival is less than c [i.e., $r(c) < c$].[6] A2(ii) ensures that the upper and lower lines of (2) are each concave in x_i, for fixed x_j. The overall payoff function F is, however, not globally concave in x_i for any fixed $x_j \in (0, c)$ and $\beta > 0$, since the left derivative of $F(x_i, x_j)$ with respect to x_i, evaluated at $x_i = x_j$, is less than the same right derivative, i.e.,

$$(1 - \beta)\tfrac{4}{9}(a - c + x_j) - \gamma x_j$$
$$< \beta \tfrac{2}{9}(a - c + x_j) + (1 - \beta)\tfrac{4}{9}(a - c + x_j) - \gamma x_j. \quad (3)$$

This means that the derivative of $F(x_i, x_j)$ with respect to x_i "jumps up" at $x_i = x_j$. Since F is not concave in x_i, the firms' reaction functions need not be continuous.

2.2. Interpretations

As described by (1), the stochastic spillover process at hand is new, and a justification is warranted. The key feature of this process is that know-how may only flow from the more R & D-intensive firm (the innovator) to the other firm (the imitator). Furthermore, the effective cost reduction of the imitator [given by X_j in (1)] is a binomial random variable with the spillover parameter β as success probability. Thus the spillover process only admits extreme realizations: Either full or no spillover will actually take place, even though β can assume any value in [0, 1]. Our stochastic spillover process considerably enlarges the scope of interpretation of the spillover effects relative to its certainty-equivalent version, thereby allowing for interesting links to the broader R & D literature, including empirical work, as we now argue.[7]

The spillover process given by (1) is a reasonable approximation for potential leakage effects in several different contexts. The first and perhaps most natural interpretation of β is as the probability that R & D leakages will actually take place. In other words, β would represent here a measure of the ease of carrying out reverse engineering efforts in the industry under consideration. This presumes the

6. Were $r(c) = c$, then it would be a dominant strategy for each firm to reduce its cost by the full amount, which is an uninteresting case.

7. The certain-equivalent version of our spillover process is given by (assuming w.l.o.g. that $x_i \geq x_j$) $X_i := x_i$ and $X_j := x_j + \beta(x_i - x_j)$, instead of (1). Here, the imitator ends up with his autonomous cost reduction plus a fraction (given by the spillover rate β) of the difference in the two cost reductions. This is simply the expected cost reduction under the original stochastic spillover process.

absence of patent protection of the differential in know-how between the two firms. Alternatively, $1 - \beta$ can be a measure of the effectiveness of patent protection.[8]

The second interpretation of β is as the *ex ante* perceived probability (by the firms) that effective patent protection will not be granted to protect the innovation differential of the leading firm. This explanation presumes an environment with potential patent protection, and an R & D process with high reverse-engineering opportunity (in the absence of patents). Some examples of granting success rates are: 65% in the US, 90% in France, 80% in the UK, but only 35% in Germany (Griliches, 1990).

The third possible interpretation of β relates it to the length of patent protection and the discount rate in a deterministic setting, with β an inversely related proxy for patent length.[9] This analogy may serve as a bridge between the literature on process R & D and that on patent races and patent design (see, e.g., Acs and Audretsch, 1989; Gilbert and Shapiro, 1990). A related alternative is to think of β as an (inversely related) measure of the imitation lag. Delays in imitation may result from a firm's organization inertia, or its inability to effectively implement internal changes. This interpretation relates our model to the business strategy literature (see Rumelt, 1995; and Spulber, 1993).

2.3. Equilibria

Before stating the proposition characterizing the equilibria of the R & D game, it is useful to define the function

$$\mathcal{I}(\beta) = 6(1-\beta) + \frac{a}{c}(2-\beta) + \sqrt{\left(6(1-\beta) - \frac{a}{c}(2-3\beta)\right)^2 + 8\beta\frac{a}{c}\left(\frac{a}{c}-2\right)(1-\beta)},$$

8. This depends on industry characteristics; it is, e.g., very high for the drug industry and low for the paper industry (Griliches, 1990).

9. With a patent for T periods, the innovator collects asymmetric per-period duopoly profits Π_A (corresponding to a larger market share) for the first T periods, and symmetric duopoly profits Π_S (corresponding to the same low unit cost for both firms) thereafter. If firms discount future profits at a rate $\delta < 1$, the present value of the innovator's profit stream is

$$\frac{\delta^T}{1-\delta}\Pi_S + \frac{1-\delta^T}{1-\delta}\Pi_A.$$

Multiplying by $1-\delta$ and comparing the outcome with the top line in (2) leads to the identification $\beta = \delta^T$.

which plays a central role in distinguishing whether the game's equilibria are interior or boundary. It should be noted that in AJ, as well as in the many follow-up studies inspired by this pioneering paper, an implicit assumption of equilibrium interiority was made throughout.

PROPOSITION 1: *Suppose A1 and A2 hold and $\beta > 0$. Then the R & D game has a unique pair of pure-strategy Nash equilibria of the form $(\bar{x}, \underline{x})$ and $(\underline{x}, \bar{x})$ with $\bar{x} > \underline{x}$. If $9\gamma > \mathcal{I}(\beta)$, then*

$$\bar{x} = (a - c)\frac{4(1 - \beta)(9\gamma - 12 + 8\beta) + 18\beta\gamma}{D} < c,$$
$$\underline{x} = (a - c)\frac{4(1 - \beta)(9\gamma - 12 + 8\beta)}{D} > 0, \tag{4}$$

where $D \triangleq [9\gamma - 8(1 - \beta)](9\gamma - 8 + 6\beta) - 16(1 - \beta)^2$. *Otherwise,* $\bar{x} = c$ *and* $\underline{x} = 4(1 - \beta)(a - 2c)/[9\gamma - 8(1 - \beta)] > 0$.

Hereafter, we use the statement that the equilibrium pair of R & D decisions $(\bar{x}, \underline{x})$ is interior and the statement $9\gamma > \mathcal{I}(\beta)$ interchangeably.

A crucial feature of our model is the asymmetry of its equilibria. Driving the asymmetry is the nonconcavity of the profit function along the diagonal where $x_1 = x_2$. To see this, note that a necessary condition for $x \in (0, c)$ to be a best response to itself is that the left derivative of $F(x_i, x_j)$ with respect to x_i, evaluated at $x_i = x_j = x$, is nonnegative and that the same right derivative is nonpositive, i.e.,

$$(1 - \beta)\tfrac{4}{9}(a - c + x) - \gamma x$$
$$\geq 0 \geq \beta\tfrac{2}{9}(a - c + x) + (1 - \beta)\tfrac{4}{9}(a - c + x) - \gamma x.$$

But this is ruled out by (3). Moreover, $a > c$ implies that $r(0) \neq 0$, and as we have already noted, $r(c) < c$. Thus $r(x) \neq x$ for any $x \in [0, c]$, i.e., if a firm's rival reduces its cost by x, it can never be a best response for the firm to conduct the same amount of R & D—it either does more or less. Amir and Wooders (1997a) shows that this asymmetry property carries over under general conditions.

We conclude this subsection by providing a partition of the parameter space according to the type of equilibrium that obtains. The dashed line in Figure 1 is the graph of the equation $9\gamma = \mathcal{I}(\beta)$. The solid line is the graph of $9\gamma = \max\{4(a/c)(1 - \beta), 8 - 6\beta\}$ and shows where A2 fails.

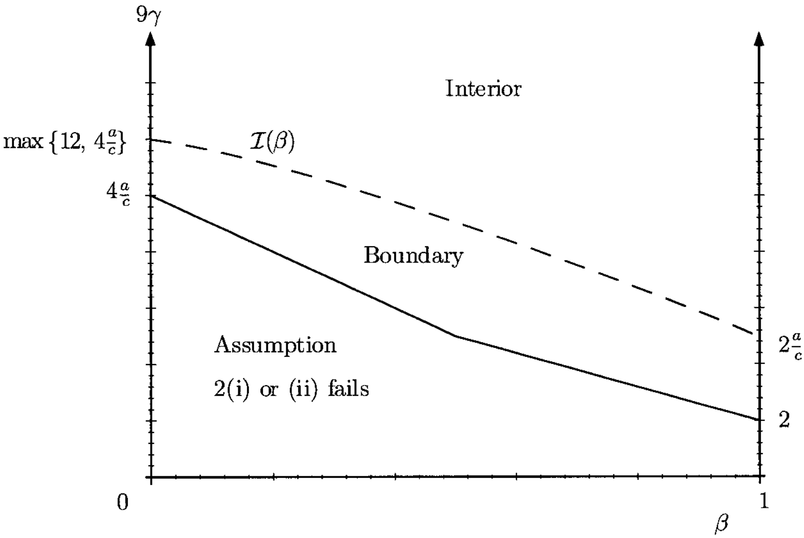

FIGURE 1.

2.4. Interfirm Heterogeneity

We now turn to the question of how R & D decisions and two measures of firm size—a firm's market share and its share of industry R & D expenditures—depend on the spillover rate and the cost of R & D. We also compare the equilibrium profit of the innovator and imitator, and consider how profits vary in β for the extreme cases when R & D is fully appropriable and when R & D is a public good.

Previous studies of R & D competition with spillovers have focused on symmetric equilibria. In this case, in a duopoly, say, the equilibrium level of R & D is given by the intersection of the reaction function with the 45° line. Hence if each firm's reaction function shifts down as β increases, the equilibrium level of R & D is decreasing in β (see, for example, KMZ). Although, in the present context, reaction functions also shift down as β increases, the analysis of the comparative statics is complicated by the asymmetry of the equilibria. Two effects combine to determine whether a firm's R & D decision is increasing or decreasing in β. The first effect is that, holding its rival's R & D fixed, a firm does less R & D as the likelihood of a spillover increases.[10] For the innovator this follows because the

10. In other words, reactions functions shift down as β increases. This follows from the submodularity of each player's payoff function, as given by (2), in own decision and β. See the proof of Proposition 1.

marginal benefit of R & D decreases as the results become less likely to be appropriable. Similarly, the marginal benefit to the imitator of his own R & D decreases as it becomes more likely he can free-ride on the innovator's R & D. The second and opposing effect is that each firm's best response to a decrease in its rival's R & D is an increase in its own R & D. The overall effect is determined by which of these effects dominates.

Surprisingly, for each firm the second effect sometimes dominates. For the innovator this means that his R & D decision sometimes increases in the likelihood of a spillover or, interpreting β as an inversely related proxy for patent length or imitation lag (see Sect. 2.2), it increases as the patent length or imitation lag shortens.[11] Proposition 2 fully characterizes these comparative statics.

PROPOSITION 2: *Let $\overline{R}$ and $\underline{R}$ be as indicated in Figure 2. (The inequalities defining these regions are provided in the proposition's proof.) Assume the equilibrium pair of R & D decisions $(\bar{x}, \underline{x})$ is interior. Then*

(i) *the innovator's R & D decision $\bar{x}$ is increasing (decreasing) in β if $(\beta, 9\gamma)$ is in the interior (the complement) of $\overline{R}$, and*
(ii) *the imitator's R & D decision $\underline{x}$ is increasing (decreasing) in β if $(\beta, 9\gamma)$ is in the interior (the complement) of $\underline{R}$.*

The innovators R & D is increasing in β in $\overline{R}$, since in $\overline{R}$ the imitator's R & D is rapidly decreasing in β, so that of the two effects mentioned above, the second dominates.

The more intuitive conclusion that each firm's R & D decision is decreasing in β is obtained with the additional assumption that demand is high relative to initial unit costs. This ensures that the graph of $9\gamma = \mathcal{I}(\beta)$, represented by the dashed line in Figure 2 (which shifts upward as a/c rises), lies above both $\overline{R}$ and $\underline{R}$ (whose positions do not depend on a or c). As well, each firm's R & D decision is everywhere decreasing in β if R & D costs are sufficiently convex. The result, stated precisely, is the following.

COROLLARY 3: *Assume the equilibrium pair of R & D decisions $(\bar{x}, \underline{x})$ is interior. If $4a/c \geq 16$, then both R & D decisions $\bar{x}$ and $\underline{x}$ are decreasing in β. Further, the innovator's R & D decision $\bar{x}$ (imitator's R & D decision $\underline{x}$) is decreasing in β if $9\gamma \geq 16$ $(9\gamma \geq 12)$.*

11. A similar result is reported by Cadot and Lippman (1995) in a different setting. Furthermore, this result is consistent with the empirical finding that the propensity to patent (patents per R & D dollar) does not exhibit any clear interindustry patterns (Griliches, 1990, p. 1678; Scherer, 1983). A measure of this concept here is $(1 - \beta)/(\delta x^2/2)$ which, according to Proposition 2, is not monotone in β.

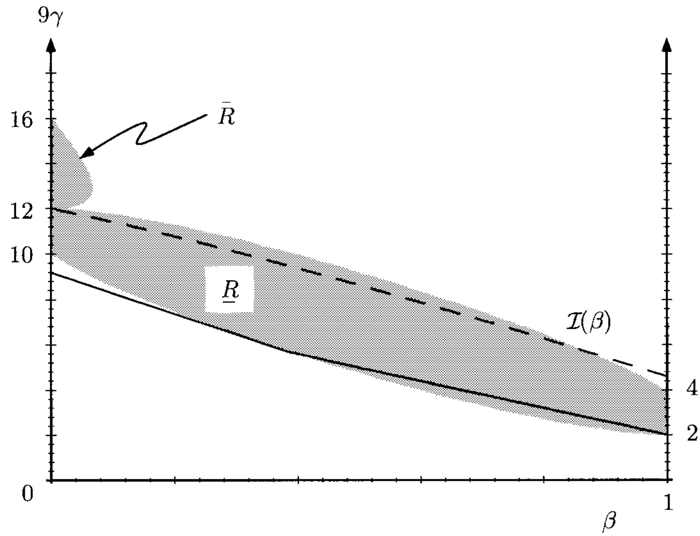

FIGURE 2.

It is of interest to provide some quantitative insight into the effects of the heterogeneous R & D behavior on market shares. In the event of a spillover, each firm has a market share of $\frac{1}{2}$. The next proposition characterizes the comparative statics of the innovator's market share in the event no spillover occurs.

PROPOSITION 4: *Assume the equilibrium pair of R & D decisions* $(\bar{x}, \underline{x})$ *is interior. Then the innovator's market share condition on the event that no spillover occurs, given by* $(a - c + 2\bar{x} - \underline{x})/[2(a - c) + \bar{x} + \underline{x}]$, *is increasing (decreasing) in* $\beta \in [0, 1)$ *if* $9\gamma > 12$ $(9\gamma < 12)$. *The innovator's market share is decreasing in* γ, *and as* γ *tends to infinity, his market share is* $\frac{1}{2}$ *(for all* $\beta \in [0, 1)$).

As indicated earlier, β can be interpreted as an inversely related proxy of the patent length and/or imitation lag. Assuming an annual discount factor of $\delta = 0.97$, a value for β of $\frac{1}{2}$ corresponds to a patent length or imitation lag of $T = 22.75$ years (i.e., $\delta^T = \beta$). With 9γ equal to 14, for example, the innovator's market share is 60% for 22.75 years, but falls to 50% once his innovation is imitated.

A measure of firm heterogeneity that depends on R & D expenditures is useful since heterogeneity in this dimension does not depend on whether or not a spillover occurs. Next, we consider how the innovator's share of total industry R & D expenditures varies in the likelihood of a spillover and the cost of R & D.

PROPOSITION 5: *Assume the equilibrium pair of R & D decisions* $(\bar{x}, \underline{x})$ *is interior. Then the innovator's share of industry R & D expenditures, given by* $\bar{x}^2/(\bar{x}^2 + \underline{x}^2)$, *is increasing (decreasing) in* $\beta \in [0, 1)$ *if* $9\gamma > 12 - 8\beta^2$ (*if* $9\gamma < 12 - 8\beta^2$). *The share is decreasing in* γ, *and in the limit as* γ *approaches infinity it is* $(2-\beta)^2/[(2-\beta)^2 + 4(1-\beta)^2]$.

Typically the innovator's share of industry R & D expenditures is increasing in β, since the interiority of equilibrium R & D decisions [i.e., $9\gamma > \mathcal{I}(\beta)$] implies $9\gamma > 12 - 8\beta^2$, unless the level of demand relative to initial unit costs is very low. Since the innovator's share of R & D expenditures is decreasing in γ and is $(2-\beta)^2/[(2-\beta)^2 + 4(1-\beta)^2]$ in the limit, this limit is provides a lower bound for the innovator's share of R & D expenditures.

Another dimension in which firms differ is their level of expected profit. When $\beta = 1$, the imitator's profit is clearly higher than the innovator's, since he receives the full benefit of the innovator's cost reduction while incurring none of the costs. For $\beta < 1$, the profit comparison is not straightforward: while the innovator has a higher expected profit at the second stage than the imitator, he also has large R & D costs at the first stage. Our next proposition provides a partition of the parameter space according to which firm earns higher expected profits.

PROPOSITION 6: *Assume the equilibrium pair of R & D decisions* $(\bar{x}, \underline{x})$ *is interior and* $\beta > 0$. *The ranking of the innovator's and the imitator's expected profit,* $F(\bar{x}, \underline{x})$ *and* $F(\underline{x}, \bar{x})$, *respectively, is given by Figure* 3. *If* $\beta \geq \frac{2}{3}$, *the imitator obtains a higher expected profit than the innovator.*

Although it is intuitive that the spillover parameter is a key determinant of the profit comparison, the fact that R & D costs play a role only for a very limited range of values of β is rather surprising. This result may be related to the debate in the eighties about innovation and profitability in international duopolies involving, e.g., US and Japanese firms in the roles of innovators and imitators, respectively.

Intuitively, one might expect that the spillover rate most favorable to the innovator is $\beta = 0$ (i.e., R & D is perfectly appropriable), and that the spillover rate most favorable to the imitator is $\beta = 1$ (i.e., R & D is a pure public good). The next proposition shows this intuition is never correct for the innovator, and not always correct for the imitator.

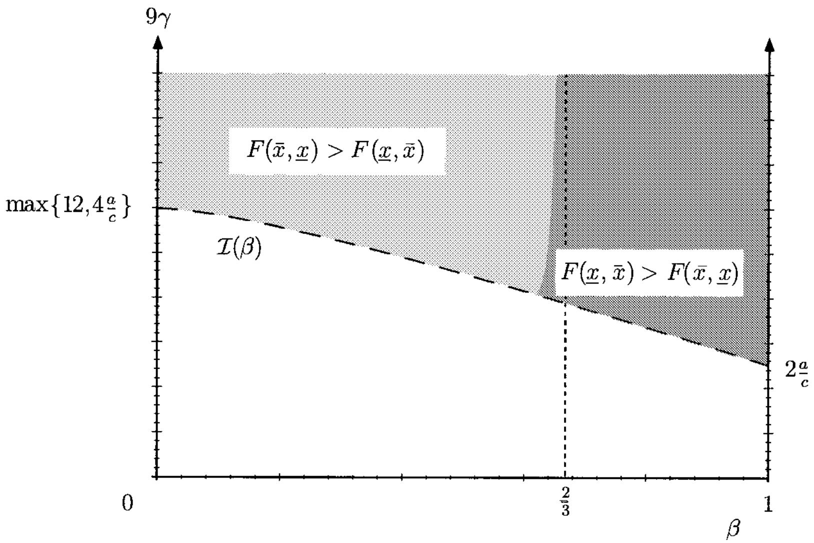

FIGURE 3.

Proposition 7: *Assume the equilibrium pair of R & D decisions* ($\bar{x}$, $\underline{x}$) *is interior. If* $\beta = 0$, *then* (i) *the innovator's equilibrium expected profit is increasing in* β, *and* (ii) *the imitator's expected profit is increasing (decreasing) in* β *if* $9\gamma > 16$ ($9\gamma < 16$). *If* $\beta = 1$, *then* (i) *the innovator's profit is decreasing in* β, *and* (ii) *the imitator's profit is increasing (decreasing) in* β *if* $9\gamma > 6 + 2\sqrt{7} \approx 11.23$ ($9\gamma < 6 + 2\sqrt{7}$).

An implication of this proposition is that unless R & D costs are low, there is a positive spillover probability for which *both* firms obtain higher expected profits than they would if R & D were perfectly appropriable. Interpreting β as an inverse proxy of patent length or imitation lag ($\beta = 0$ corresponds to an infinite patent length or imitation lag, while $\beta = 1$ corresponds to patent length or imitation lag of zero length), this means there is a finite patent length that both firms prefer to an infinite patent length.

The intuition underlying this result is clear once one observes that there are two opposing effects on the innovator's equilibrium profit when β increases from zero. The first is the effect on the innovator's profits of the resulting change in R & D intensity by the imitator. When the equilibrium pair of R & D decisions ($\bar{x}$, $\underline{x}$) is interior and $\beta = 0$ [i.e., $9\gamma > \mathcal{I}(0)$], this implies $9\gamma > 12$, and thus the imitator reduces his R & D intensity as β increases (see Corollary 3).

In this case the first effect is positive, since the innovator faces a weaker competitor in the output market when no spillover occurs. Opposing this effect is the negative effect on the innovator's profit of an increase in the likelihood his R & D results spill to his rival. This effect vanishes at $\beta = 0$, since, when a spillover is certain not to occur, both firms conduct the same R & D. Thus the innovator's profit is increasing in β at $\beta = 0$. Similar reasoning shows that the imitator's profit is increasing (decreasing) in β at $\beta = 0$ if the innovator's R & D is decreasing (increasing) in β, i.e., if $9\gamma > 16$ ($9\gamma < 16$).

3. Research Joint Ventures

This section compares R & D competition and R & D cooperation via a joint lab. Under this form of cooperation, R & D is conducted in a single jointly owned lab, costs are shared equally by the firms, and results are fully communicated to both firms. The performance criteria of interest are: R & D propensity, individual (and total) firm profits, consumer surplus, and social welfare.

The joint lab chooses a cost reduction x that maximizes the sum of the firms' profits when R & D results are fully communicated to both firms, i.e., the joint lab solves

$$\max_{x \in [0, c]} \left(\tfrac{2}{9}(a - c + x)^2 - \frac{\gamma}{2} x^2 \right). \tag{5}$$

The solution to (5), denoted by x_J, is $x_J = 4(a - c)/(9\gamma - 4)$ if $9\gamma > 4a/c$, and $x_J = c$ otherwise.

The joint lab as a form of R & D cooperation is of particular interest in that, when spillovers are one-way, the joint lab is equivalent to KMZ's case *CJ*, or cartelized RJV, in which firms coordinate R & D expenditures in separately owned labs and fully communicate R & D results. In KMZ a cartelized RJV dominates R & D competition, as well as any other form of R & D cooperation in which the firms either coordinate R & D decisions or communicate R & D results, for each of the performance criteria of interest (assuming symmetric outcomes throughout).

We begin with a comparison of R & D propensities. According to the next Proposition, the imitator always conducts less R & D than the joint lab. The innovator conducts less R & D than the joint lab if either R & D costs are sufficiently convex or demand relative to initial costs is sufficiently high. The innovator conducts more R & D than the joint lab if R & D costs are not too convex and demand is not too high.

Proposition 8: *Assume $\beta > 0$. The ranking of $\bar{x}$, $\underline{x}$, and x_J is given in Figure 4. The imitator always conducts less R & D than the joint lab, while the innovator may conduct either more or less: $x_J \geq \bar{x}$ if either* (i) $9\gamma \geq 16(1-\beta)$ *or* (ii) $9\gamma \leq 4a/c$, *but* $x_J < \bar{x}$ *if* $4a/c < 9\gamma < 16(1-\beta)$. *Further,* $4a/c \geq 16$ *implies either* $9\gamma \geq 16(1-\beta)$ *or* $9\gamma \leq 4a/c$ *and hence* $x_J \geq \bar{x}$.

Our next result concerns the comparison of equilibrium total profits, showing that total profits are higher under the joint lab if either (i) demand is sufficiently high relative to initial unit costs, (ii) R & D costs are sufficiently convex, or (iii) the equilibrium is interior and R & D costs satisfy a weak convexity condition.

Proposition 9: *Total profits are higher under a joint lab than under R & D competition if any one of the following conditions holds*:

(i) $a/c \geq \frac{5}{2}$,
(ii) $9\gamma \geq 18$, *or*
(iii) *the equilibrium pair of R & D decisions* $(\bar{x}, \underline{x})$ *is interior and* $9\gamma \geq 12 - 7\beta$.

The proposition establishes the superiority of the joint lab, in terms of total profits, under general conditions. As the following example shows, when none of the conditions of the proposition is satisfied, the sum of profits may be higher under R & D competition.

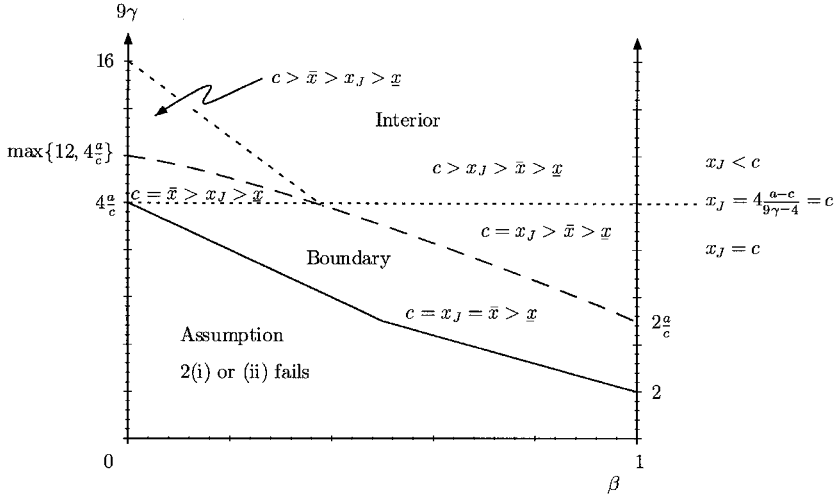

FIGURE 4.

EXAMPLE 1: Let $9\gamma = 4.5$, $a = 210$, $c = 100$, and $\beta = 0.964$. Then $\bar{x} = 99.768$, $x = 0.34982$, and $x_J = 100$. Under R & D competition, the innovator's and imitator's profits are, respectively, $F(\bar{x}, \underline{x}) = 2607.1$ and $F(\underline{x}, \bar{x}) = 4713.6$, for a combined profit of 7320.7. Total profits under the joint lab are only $\frac{2}{9}(210)^2 - (0.5/2)(100)^2 = 7300$.

In light of the obvious advantages of the joint lab—R & D decisions are coordinated to maximize the sum of profits, and R & D results are fully shared, thereby avoiding duplication of effort—it is natural to ask how the sum of profits can be higher under R & D competition. The answer is that R & D competition conveys the (potential) advantage that the firms compete asymmetrically in the second stage, while in a joint lab symmetry at the second stage is built in.

To see that asymmetry can be advantageous, consider the hypothetical problem in which a RJV has the know-how to reduce costs by $k \le c$, and this know-how is to be distributed to the two firms in order to maximize the sum of profits. Denoting by x_i the know-how in cost reduction distributed to firm i, the RJV's problem is

$$\max_{0 \le x_1, x_2 \le k} \left[\tfrac{1}{9}(a - c + 2x_1 - x_2)^2 + \tfrac{1}{9}(a - c + 2x_2 - x_1)^2 \right].$$

The objective function is strictly convex in x_1 and x_2. It is straightforward to verify that the solution has maximal differentiation (i.e., $x_i = 0$ and $x_j = k$, $i \ne j$) if $3k - 2(a - c) \ge 0$, and minimal differentiation (i.e., $x_1 = x_2 = k$) otherwise. When the solution has maximal differentiation, the RJV would find it advantageous to differentiate the firms. Firms are always differentiated under R & D competition, but differentiation is ruled out under the joint lab since R & D results are fully shared. Thus, combined profits are greater under R & D competition if the gains from differentiation exceed the losses from lack of coordinated R & D and duplication of effort.

We now turn our attention to comparing consumer surplus under R & D competition and R & D cooperation. When comparing consumer surpluses, we are concerned with *expected* surplus, since prior to the realization of spillovers the output is stochastic. Given an autonomous cost reductions x_i and x_j, with $x_i \ge x_j$, say, the expected consumer surplus is

$$V(x_i, x_j) = \beta \frac{1}{2}\left(\frac{2(a - c) + 2x_i}{3}\right)^2 + (1 - \beta)\frac{1}{2}\left(\frac{(2a - c) + x_i + x_j}{3}\right)^2. \tag{6}$$

When the innovator conducts more R & D than the joint lab then, when a spillover occurs, the total output is higher and the price is lower under R & D competition. Moreover, even were expected output under R & D competition and the joint lab the same, consumer surplus would be higher under R & D competition, since it is a convex function of output.[12] In light of these facts, it is not obvious that R & D cooperation via a joint lab dominates R & D competition from the point of view of consumer surplus. Nonetheless, we have the following result.

Proposition 10: *Assume that $\beta > 0$ and the equilibrium pair of R & D decisions $(\overline{x}, \underline{x})$ is interior. Expected consumer surplus is higher under the joint lab than under R & D competition.*

Using Proposition 10 it is possible to compare the expected total cost reduction, the expected total output, and the expected price under R & D competition and under the joint lab. Expected total cost reduction with R & D competition is $(1 + \beta)\overline{x} + (1 - \beta)\underline{x}$, which we denote by $\hat{x}$. We can write expected consumer surplus under the joint lab and under R & D competition as $V(x_J, x_J)$ and $V(\overline{x}, \underline{x})$, respectively, and by Proposition 10 we have $V(x_J, x_J) > V(\overline{x}, \underline{x})$ for $\beta > 0$. Since surplus is a convex function of output (and hence also of total cost reduction), the expected surplus under R & D competition is higher than it would be were each firm to have for certain any cost reductions that sum to $\hat{x}$. It is true, in particular, for $\beta > 0$ that $V(\overline{x}, \underline{x}) > V(\hat{x}/2, \hat{x}/2)$, and thus $V(x_J, x_J) > V(\hat{x}/2, \hat{x}/2)$ for $\beta > 0$. Since $V(x, x)$ is an increasing function of x, we have $2x_J > \hat{x}$, which establishes the following corollary.

Corollary 11: *Assume that $\beta > 0$ and the equilibrium pair of R & D decisions $(\overline{x}, \underline{x})$ is interior. Then under R & D competition,*

(i) *the expected total cost reduction is lower than under the joint lab, i.e.,* $(1 + \beta)\overline{x} + (1 - \beta)\underline{x} < 2x_J$,
(ii) *the expected total output is lower than under the joint lab, and*
(iii) *the expected price is higher than under the joint lab.*

Parts (ii) and (iii) of Corollary 11 follow from the fact that output is positively and linearly related to total cost reduction.

12. When total output is $q_1 + q_2$, the consumer surplus is $\frac{1}{2}(q_1 + q_2)^2$. Thus consumers prefer a lottery on total output to the associated average output with certainty.

4. Optimal R & D Cartels

In this section we characterize the structure of R & D cartels when, in addition to choosing (possibly asymmetric) cost reductions, a cartel also chooses the spillover rate in $[0,1]$. An R & D cartel is described by a triple (x_1, x_2, s), where x_i is the cost reduction of firm i and s is the spillover rate. The expected profit of the cartel (x_1, x_2, s), denoted by $C(x_1, x_2, s)$, is

$$s\tfrac{2}{9}(a-c+x_1 \vee x_2)^2 + (1-s)\Big[\tfrac{1}{9}(a-c+2x_1-x_2)^2 + \tfrac{1}{9}(a-c+2x_2-x_1)^2\Big] - \frac{\gamma}{2}(x_1^2+x_2^2).$$

An optimal cartel (x_1^*, x_2^*, s^*) satisfies

$$(x_1^*, x_2^*, s^*) \in \arg \max_{x_1, x_2 \in [0,c],\, s \in [0,1]} C(x_1, x_2, s).$$

Allowing the firms to choose a spillover rate below the natural β can be interpreted as letting the firms locate farther away from one another, agreeing not to hire each other's scientists, etc. If it is not possible to choose a spillover rate below its natural rate, the proposition's proof makes it clear that whenever 0 is preferred to 1, then $s=\beta$ would be preferred to $s=1$ (due to the linearity of C in β). In this case an R & D cartel would simply keep the natural rate β.

Proposition 12: *Under A1, we have*

(i) *The optimal R & D cartel (x_1^*, x_2^*, s^*) has $x_1^* \neq x_2^*$, and either full or no spillovers, i.e., $s^*=0$ or $s^*=1$, provided $a/c \neq \frac{5}{2}$.*
(ii) *If either $9\gamma > 18$ or $a/c > \frac{5}{2}$, then the optimal cartel is $(x_1^*, x_2^*, s^*) = (x_J, 0, 1)$.*
(iii) *If $9\gamma < 4a/c$ and $a/c < \frac{5}{2}$, then the optimal cartel is $(x_1^*, x_2^*, s^*) = (c, 0, 0)$.*

The intuition behind this proposition is best described in terms of a tension between two conflicting effects. The first is an efficiency effect, that identical Courtnot rivals' profits increase as the (common) unit cost declines, thus pushing for the choice $s^*=1$. The second effect is the joint desire for cost asymmetry discussed earlier, which is best achieved under a no-spillover regime. Under this perspective, the proposition simply identifies specific conditions for each of the two effects to be dominant.

R & D cartelization may thus also lead to asymmetries among firms under one-way spillovers. This point was first made by Salant and Shaffer (1998) in their perceptive note on the AJ and KMZ models. Interestingly, these models and ours coincide for an R & D cartel when spillovers are either 0 or 1. (Case N differs across the three models when $\beta = 1$.) Thus our results here also shed some light on the optimal R & D cartel for the other models, an important issue that has not been addressed in the literature on R & D cooperation so far.

5. Extensions

In view of the central role of the spillover process in this study, a robustness analysis showing that the key feature of equilibrium asymmetry would survive under more general one-way spillover processes is warranted. Consider the case where β depends on $|x_i - x_j|$, with $\beta'(\cdot) \leq 0$, so that the spillover probability increases as the imitator closes the gap in R & D levels.[13] In the firm's payoff (2), one would need to replace β by $\beta(x_i - x_j)$ in the top line and by $\beta(x_j - x_i)$ in the bottom line. Call the resulting expressions $U(x_i, x_j)$ and $L(x_i, x_j)$, respectively. As argued around (3), a sufficient condition to rule out symmetric equilibria is $\partial U/\partial x_i > \partial L/\partial x_i$ along the diagonal $x_i = x_j$. To this end, it suffices to have $\beta(0) > 0$ (the details are omitted). A simple example is $\beta(|x_i - x_j|) = \alpha(1 - |x_i - x_j|/c)$, where $\alpha \in (0, 1]$ represents the maximal spillover probability. In conclusion, the asymmetry property is robust to the specification of one-way spillovers. On the other hand, it is clear that the linear-quadratic structure of the model would not survive such a generalization, and thus analytic tractability would be lost. Furthermore, some other properties of our model, such as concavity and submodularity of U and L, would require other assumptions on $\beta'(\cdot)$ and $\beta''(\cdot)$.

6. Conclusion

With one-way spillovers, the standard symmetric two-period R & D model leads to asymmetric equilibria only. This links free-rider effects in R & D to the emergence of intra-industry heterogeneity. The

13. This would nicely capture the notion that a lagging firm's absorptive capacity in R & D know-how increases with its R & D spending. Note that all previous related studies considered constant spillover rates. We are grateful to an anonymous referee for suggesting this extension.

latter is extensively characterized, with emphasis on the effects of the spillover rate on market shares, profits, and R & D levels.

We also find that the innovator sometimes conducts more R & D than the joint lab, and the sum of profits is sometimes higher under R & D competition than under the joint lab. To recover AJ's and KMZ's conclusions here, one needs additional assumptions on the convexity of R & D costs or on the level of demand. On the other hand, we find that consumer welfare is always higher under the joint lab than under R & D competition. Thus, our results still provide support for a hands-off antitrust policy vis-à-vis RJV's.

Appendix: Proofs

The proofs of Corollary 3 and Propositions 5 and 8 are provided in Amir and Wooders (1997b), henceforth AW, as Corollary 4 and Propositions 6 and 9, respectively.

Proof of Proposition 1: AW establishes that the reaction functions have the linear character illustrated in Figure 5. Simple calculations then establish that there is a unique pair of pure-strategy Nash equilibria, $(\bar{x}, \underline{x})$ and $(\underline{x}, \bar{x})$, with $\bar{x}$ and $\underline{x}$ as given in the proposition. □

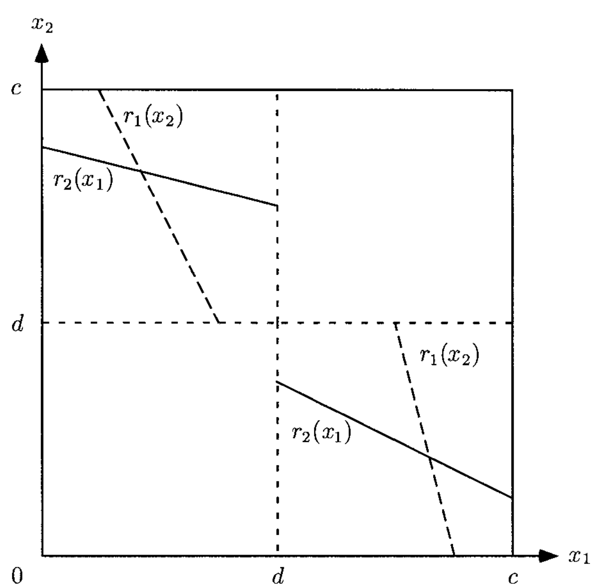

FIGURE 5.

Proof of Proposition 2: Region $\overline{R}$ is the set of $(\beta, 9\gamma)$ for which $\beta \in [0, \frac{1}{2} - \frac{1}{4}\sqrt{3})$ and

$$14 - 16\beta - 2\sqrt{1 - 16\beta + 16\beta^2} \le 9\gamma \le 14 - 16\beta + 2\sqrt{1 - 16\beta + 16\beta^2}.$$

Region $\underline{R}$ is the set of $(\beta, 9\gamma)$ such that

$$11 - 8\beta - \sqrt{1 + 16\beta - 16\beta^2} \le 9\gamma \le 11 - 8\beta + \sqrt{1 + 16\beta - 16\beta^2}.$$

Calculations in AW establish $d\overline{x}/d\beta > 0$ ($d\overline{x}/d\beta < 0$) if $(\beta, 9\gamma)$ is in the interior (the complement) of $\overline{R}$ and $d\underline{x}/d\beta > 0$ ($d\underline{x}/d\beta < 0$) if $(\beta, 9\gamma)$ is in the interior (the complement) of $\underline{R}$). □

Proof of Proposition 4: When there is no spillover *ex post*, the innovator's output is $\overline{q} = (a - c + 2\overline{x} - \underline{x})/3$, while the imitator's output is $\underline{q} = (a - c + 2\underline{x} - \overline{x})/3$. When the equilibrium is interior, the innovator's market share is

$$\frac{\overline{q}}{\overline{q} + \underline{q}} = \frac{1}{2}\left(1 + \frac{3\beta}{9\gamma - 12 + 11\beta}\right).$$

Differentiating with respect to β yields $\frac{3}{2}(9\gamma - 12)/(9\gamma - 12 + 11\beta)^2$, which is positive iff $9\gamma > 12$. Differentiating with respect to γ yields $-\frac{27}{2}\beta/(9\gamma - 12 + 11\beta)^2 < 0$. Clearly $\lim_{\gamma \to \infty} \overline{q}/(\overline{q} + \underline{q}) = \frac{1}{2}$. □

Proof of Proposition 6: Calculations in AW show that $F(\overline{x}, \underline{x}) > F(\underline{x}, \overline{x})$ in the lightly shaded region defined by the inequalities $9\gamma > \mathcal{I}(\beta)$, $0 < \beta < \frac{2}{3}$, and

$$9\gamma > \frac{2(1 - \beta)(12 - 17\beta)}{2 - 3\beta}.$$

It is also shown that $F(\underline{x}, \overline{x}) > F(\overline{x}, \underline{x})$ if $9\gamma > \mathcal{I}(\beta)$ and either (1) $0 < \beta < \frac{2}{3}$ and $9\gamma < 2(1 - \beta)(12 - 17\beta)/(2 - 3\beta)$, or (2) $\beta \ge \frac{2}{3}$. □

Proof of Proposition 7: We prove the result only for the innovator, as the same methods establish the result for the imitator. The innovator's profit is

$$F(\overline{x}, \underline{x}) = \beta\tfrac{1}{9}(a - c + \overline{x})^2 + (1 - \beta)\tfrac{1}{9}(a - c + 2\overline{x} - \underline{x})^2 - \frac{\gamma}{2}\overline{x}^2.$$

Differentiating $F(\bar{x}, \underline{x})$ totally with respect to β and simplifying after applying the envelope theorem yields

$$\frac{dF(\bar{x}, \underline{x})}{d\beta} = -\tfrac{2}{9}(1-\beta)(1+2\bar{x}-\underline{x})\frac{d\underline{x}}{d\beta}$$

$$+\tfrac{1}{9}(1+\bar{x})^2 - \tfrac{1}{9}(1+2\bar{x}-\underline{x})^2.$$

At $\beta = 0$, we have

$$\frac{dF(\bar{x}, \underline{x})}{d\beta} = -\frac{2}{9}\left(1+\frac{4}{9\gamma-4}\right)\frac{d\underline{x}}{d\beta}\bigg|_{\beta=0},$$

using that $\bar{x}|_{\beta=0} = \underline{x}|_{\beta=0} = 4/(9\gamma-4)$. Then $9\gamma > \mathcal{I}(0)$ implies $9\gamma > 12$, which can be shown to imply $d\underline{x}/d\beta|_{\beta=0} < 0$. Hence $dF(\bar{x}, \underline{x})/d\beta|_{\beta=0} > 0$. At $\beta = 1$ we have

$$\frac{dF(\bar{x}, \underline{x})}{d\beta} = \frac{1}{9}\left(1+\frac{2}{9\gamma-2}\right)^2 - \frac{1}{9}\left(1+\frac{4}{9\gamma-2}\right)^2 < 0,$$

using that $\bar{x}|_{\beta=1} = 2/(9\gamma-2)$, and $\underline{x}|_{\beta=1} = 0$. □

Proof of Proposition 9: Denote by C_s the problem where firms coordinate their R & D investments (so as the maximize total profits) while the spillover rate is s. In other words, the firms solve: $\max\{F(x_1, x_2) + F(x_2, x_1)\colon x_1, x_2 \text{ in } [0, c]\}$ with β set equal to s in (2). Assuming without loss of generality that $x_1 \geq x_2$, problem C_s is

$$\max_{x_1, x_2 \in [0,c]} \left(\tfrac{1}{9}(a-c+2x_1-x_2)^2 + \tfrac{1}{9}(a-c+2x_2-x_1)^2\right.$$

$$\left. + sD(x_1, x_2) - \frac{\gamma}{2}(x_1^2+x_2^2)\right),$$

where $D(x_1, x_2) = \frac{2}{9}(a-c+x_1)^2 - \frac{1}{9}(a-c+2x_1-x_2)^2 - \frac{1}{9}(a-c+2x_2-x_1)^2$. Denote the objective function in this maximization problem by $C_s(x_1, x_2)$. We can write the sum of profits under the joint lab as $C_1(x_J, 0)$, and under R & D competition as $C_\beta(\bar{x}, \underline{x})$. Since C_1 is the same as the problem facing the joint lab, $(x_J, 0)$ is the unique solution to C_1, and so $C_1(x_J, 0) > C_1(\bar{x}, \underline{x})$.

Proof of (i): The inequality $D(x_1, x_2) \geq 0$ can be written as $(x_1 - x_2)[2(a-c) + 5x_2 - 3x_1] \geq 0$. Since $x_2 \geq 0$ and $x_1 \leq c$, for this inequality to hold it is sufficient that $2(a-c)-3c \geq 0$, or $a/c \geq \frac{5}{2}$. Hence $a/c \geq \frac{5}{2}$ implies $C_s(x_1, x_2)$ is nondecreasing in s. Thus $C_1(\bar{x}, \underline{x}) \geq C_\beta(\bar{x}, \underline{x})$, which together with $C_1(x_J, 0) \geq C_1(\bar{x}, \underline{x})$ implies $C_1(x_J, 0) \geq C_\beta(\bar{x}, \underline{x})$.

Proof of (ii): It can be verified that the objective in C_0 is jointly concave in (x_1, x_2) when $9\gamma \geq 18$. Since this objective is symmetric, there must be a unique arg max, which is also symmetric, i.e., of the form (x^*, x^*), with $C_0(\bar{x}, \underline{x}) \leq C_0(x^*, x^*)$. Consequently, one can restrict the maximization of the objective in C_0 to choices on the diagonal, i.e., replace the objective with $\frac{2}{9}(a-c+x_1)^2 - \gamma x_1^2$, which is clearly below the objective in C_1, i.e., $\frac{2}{9}(a-c+x_1)^2 - (\gamma/2)x_1^2$. Hence $C_0(x^*, x^*) \leq C_1(x_J, 0)$ and so $C_0(\bar{x}, \underline{x}) \leq C_1(x_J, 0)$. This inequality, together with $C_1(x_J, 0) > C_1(\bar{x}, \underline{x})$ from above, implies $C_\beta(\bar{x}, \underline{x}) = \beta C_1(\bar{x}, \underline{x}) + (1-\beta)C_0(\bar{x}, \underline{x}) < C_1(x_J, 0)$.

Proof of (iii): When the equilibrium is interior, straightforward calculations show that $9\gamma \geq 12 - 7\beta$ implies that $D(\bar{x}, \underline{x}) \geq 0$. Thus, we have $C_\beta(\bar{x}, \underline{x}) \leq C_1(\bar{x}, \underline{x})$, which together with $C_1(\bar{x}, \underline{x}) < C_1(x_J, 0)$ yields $C_\beta(\bar{x}, \underline{x}) < C_1(x_J, 0)$, which is the desired result. □

Proof of Proposition 10: If $x_J \geq \bar{x} > \underline{x}$ then it is obvious that $V(x_J, x_J) \geq V(\bar{x}, \underline{x})$. We now show $V(x_J, x_J) \geq V(\bar{x}, \underline{x})$ even if $\bar{x} > x_J$. The difference $V(x_J, x_J) - V(\bar{x}, \underline{x})$ is given by

$$\frac{1}{2}\left(\frac{2(a-c)+2x_J}{3}\right)^2$$

$$-\left[\beta\frac{1}{2}\left(\frac{2(a-c)+2\bar{x}}{3}\right)^2 + (1-\beta)\frac{1}{2}\left(\frac{2(a-c)+\bar{x}+\underline{x}}{3}\right)^2\right].$$

Calculations in AW establish that this difference is positive. □

Proof of Proposition 12: It is convenient to rewrite the objective function $C(x_1, x_2, s)$ as

$$\frac{1}{9}(a-c+2x_1-x_2)^2 + \frac{1}{9}(a-c+2x_2-x_1)^2$$

$$+ sD(x_1, x_2) - \frac{\gamma}{2}(x_1^2 + x_2^2),$$

where $D(x_1, x_2) = \frac{2}{9}(a-c+x_1 \vee x_2)^2 - \frac{1}{9}(a-c+2x_1-x_2)^2 - \frac{1}{9}(a-c+2x_2-x_1)^2$. Let (x_1^*, x_2^*, s^*) be an optimal R & D cartel.

Proof of (i): We have $C(x, x, s^*) < C(x, 0, 1)$ if $x > 0$, and $C(x, x, s^*) < C(x_J, 0, 1)$ if $x = 0$, and hence $x_1^* \neq x_2^*$. Suppose contrary to (i) that $0 < s^* < 1$. It must be the case that $D(x_1^*, x_2^*) = 0$, since $D(x_1^*, x_2^*) > 0$ with $s^* < 1$ implies $C(x_1^*, x_2^*, 1) > C(x_1^*, x_2^*, s^*)$, and $D(x_1^*, x_2^*) < 0$ with $s^* > 0$ implies $C(x_1^*, x_2^*, 0) > C(x_1^*, x_2^*, s^*)$, a contradiction in either case.

Since $C(x_1, x_2, s) = C(x_2, x_1, s)$, it is without loss of generality to assume $x_1^* > x_2^*$. Then $D(x_1^*, x_2^*) = 0$ can be written as

$$\tfrac{1}{9}(x_1^* - x_2^*)(2(a-c) + 5x_2^* - 3x_1^*) = 0. \tag{7}$$

Since $D(x_1^*, x_2^*) = 0$, we have $C(x_1^*, x_2^*, 0) = C(x_1^*, x_2^*, s^*) = C(x_1^*, x_2^*, 1)$. Clearly $x_2^* = 0$, since otherwise $C(x_1^*, x_2^*, 1) < C(x_1^*, 0, 1)$ and hence $C(x_1^*, x_2^*, s^*) < C(x_1^*, 0, 1)$, a contradiction. Hence $x_1^* - x_2^* > 0$, which implies $x_1^* = 2(a-c)/3$ by (7). This fact and $a/c \neq \frac{5}{2}$ imply $x_1^* \neq c$, and so $x_1^* \in (0, c)$.

It must be that $x_1^* \in \arg\max_{x \in [0,c]} C(x, x_2^*, 0)$ and $x_1^* \in \arg\max_{x \in [0,c]} C(x, x_2^*, 1)$. [If $x_1^* \notin \arg\max_{x \in [0,c]} C(x, x_2^*, 0)$, for example, then there is an x' such that $C(x', x_2^*, 0) > C(x_1^*, x_2^*, 0) = C(x_1^*, x_2^*, s^*)$, contradicting that (x_1^*, x_2^*, s^*) is an optimal cartel.] Then $\partial C(x_1, x_2, s)/\partial x_1|_{(x_1^*, x_2^*, 0)} = 0$ implies $x_1^* = 2(a-c)/(9\gamma - 10)$, while $\partial C(x_1, x_2, s)/\partial x_1|_{(x_1^*, x_2^*, 1)} = 0$ implies $x_1^* = 4(a-c)/(9\gamma - 4)$. These two expressions for x_1^* imply $9\gamma = 16$, and hence $x_1^* = (a - c)/3$, but this contradicts $x_1^* = 2(a-c)/3$.

Proof of (ii): We show that either $9\gamma > 18$ or $a/c > \frac{5}{2}$ implies $D(x_1^*, x_2^*) > 0$, and therefore $s^* = 1$. The optimal cartels are then $(x_J, 0, 1)$ and $(0, x_J, 1)$, since the maximizers of $C(x_1, x_2, 1)$ are $(x_J, 0)$ and $(0, x_J)$. First we show that $9\gamma > 18$ implies $D(x_1^*, x_2^*) > 0$. Suppose to the contrary that $D(x_1^*, x_2^*) \leq 0$, and therefore $C(x_1^*, x_2^*, s^*) \leq C(x_1^*, x_2^*, 0)$. Since $9\gamma > 18$, we have that $C(x_1, x_2, 0)$ is jointly (strictly) concave in x_1 and x_2, and so $x_1^* \neq x_2^*$ implies

$$C\left(\frac{x_1^* + x_2^*}{2}, \frac{x_1^* + x_2^*}{2}, 0\right) > C(x_1^*, x_2^*, 0) \geq C(x_1^*, x_2^*, s^*),$$

which is a contradiction.

Next we show that $a/c > \frac{5}{2}$ implies $D(x_1^*, x_2^*) > 0$. Assume without loss of generality that $x_1^* > x_2^*$. That $x_2^* \geq 0$ and $x_1^* \leq c$ implies $2(a-c) + 5x_2^* - 3x_1^* \geq 2(a-c) - 3c = 2a - 5c > 0$, since $a/c > \frac{5}{2}$. Therefore $D(x_1^*, x_2^*) > 0$ by (7).

Proof of (iii): By part (i) either $s^* = 0$ or $s^* = 1$. We show that $9\gamma < 4a/c$ and $a/c < \frac{5}{2}$ implies $\max_{x_1, x_2 \in [0,c]} C(x_1, x_2, 0) > \max_{x_1, x_2 \in [0,c]} C(x_1, x_2, 1)$, i.e., $s^* = 0$. Since $9\gamma < 4a/c$ and $a/c < \frac{5}{2}$, then $9\gamma < 10$, and hence the objective $C(x_1, x_2, 0)$ is jointly strictly convex in (x_1, x_2), and thus it is maximized on *corners* of $[0, c]^2$, i.e., its maximizers are among $(c, 0)$, $(0, c)$, $(0, 0)$, and (c, c). We have $C(c, 0, 0) = C(0, c, 0) = (a - c + 2c)^2/9 + (a - 2c)^2/9 - (\gamma/2)c^2$. It is straightforward to show that if $9\gamma \leq 4a/c$, then the maximizers of $C(x_1, x_2, 1)$ are $(c, 0)$ and $(0, c)$. We have $C(c, 0, 1) = \frac{2}{9}a^2 - (\gamma/2)c^2$. A simple calculation establishes that $a/c < \frac{5}{2}$ implies $C(c, 0, 0) > C(c, 0, 1)$, and hence $s^* = 0$. Furthermore $C(x, x, 0) < C(c, 0, 1)$ for $x = 0$ and $x = c$ implies the optimal cartels are $(c, 0, 0)$ and $(0, c, 0)$. □

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