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Games and Economic Behavior 42 (2003) 183–207

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**GAMES and  
Economic  
Behavior**

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## Noncooperative versus cooperative R&D with endogenous spillover rates

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Received 5 July 2000

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### Abstract

This paper deals with a general version of a two-stage model of R&D and product market competition. We provide a thorough generalization of previous results on the comparative performance of noncooperative and cooperative R&D, dispensing in particular with ex-post firm symmetry and linear demand assumptions. We also characterize the structure of profit-maximizing R&D cartels where firms competing in a product market jointly decide R&D expenditure, as well as internal spillover, levels. We establish the firms would essentially always prefer extremal spillovers, and within the context of a standard specification, derive conditions for the optimality of minimal spillover.

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*JEL classification:* C72; L13; O31

*Keywords:* Oligopolistic R&D; Endogenous spillovers; Research joint ventures; R&D cartel

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### 1. Introduction

Over the last decade, the economics of research joint ventures (or RJs) has emerged as one of the most active fields of research in industrial economics. Prior to that, Tirole (1988) wrote “Very little work has been done on the subject of RJs, which is surprising in view of their potential importance in the antitrust area (particularly for high technology industries).” This rather sudden surge of interest in R&D cooperation seems to have been

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triggered by the passage of the National Cooperation Act in the United States in 1984, allowing firms to cooperate in R&D provided they remain competitors in product markets. Indeed, although a permissive antitrust attitude towards R&D cooperation has been the norm in Europe and Japan early on, no theoretical or empirical inquiry into this important issue was available to guide the antitrust decision process before the pioneering, and unfortunately mostly unnoticed, work by Ruff (1969).<sup>1</sup>

More recently, following work by Katz (1986) and d'Aspremont and Jacquemin (1988), henceforth AJ, this theoretical omission was promptly addressed by many researchers: see DeBondt (1997) for an extensive survey. In particular, Kamien et al. (1992), henceforth KMZ, analyze a model related to AJ's and extend the analysis in a number of different directions, including a richer set of R&D cooperation scenarios, see also Kamien and Zang (1993).<sup>2</sup>

In this literature, the main focus is on a performance evaluation of various forms of cooperative R&D relative to noncooperative R&D for firms competing in a product market. The comparison criteria are standard: equilibrium levels of R&D, producer and consumer surplus, and social welfare. In most studies, R&D results are imperfectly appropriable to a degree parametrized between 0 and 1, called the spillover parameter. R&D cooperation can be realized along two distinct (but possibly simultaneous) dimensions for firms competing in a product market. First, firms may cartelize their R&D expenditure levels in order to maximize their joint product market (e.g., Cournot) profits while conducting R&D in separate labs (with the spillover parameter kept at its natural level). Second, firms may jointly agree to internally set the spillover parameter equal to its maximal value of 1, thereby fully sharing their R&D activity amongst the participants (while choosing their R&D levels noncooperatively). KMZ refers to the former scenario as an R&D cartel, to the second one as a research joint venture, and to their simultaneous implementation as a cartelized RJV.

AJ and KMZ's main result states that the cartelized RJV is superior to all the other scenarios considered, including in particular the noncooperative case, along every criterion of interest: at equilibrium, it yields the lowest price in the product market, the highest level of final R&D and profit for each firm, and the highest social welfare. Thus, at least for industries with nearly symmetric firms, this result maintains that full cooperation in R&D (along both dimensions described above) is good for everyone concerned, provided that the participating firms indeed remain competitors in the product market. The subtle and important issue of whether cooperation in R&D increases the likelihood of collusion in the product market has not received much theoretical attention so far.

The analysis of AJ, KMZ, and most of the other follow-up papers relies on a number of simplifying assumptions. The underlying two-stage game is always symmetric, i.e., the

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<sup>1</sup> As early as 1957, Article 85 of the Treaty of Rome stipulated a block exemption of R&D cooperation from antitrust restrictions. In fact, both in Europe and in Japan, RJs are often initiated in specific programs (such as ESPRIT and EUREKA in Europe and VLSI in Japan) launched by government bodies, and heavily subsidised. The 1984 Act in the USA was then seen as a response to enhance the international competitiveness of US firms in world markets.

<sup>2</sup> An interesting new strand of the literature on RJs is based on a mechanism design approach: see d'Aspremont et al. (1998, 2000).

firms are ex-ante identical. Demand and production costs are always taken to be linear. With the exception of KMZ, R&D costs are typically postulated to be quadratic in the level of cost reduction. Furthermore, for all R&D scenarios considered, attention has always been restricted to symmetric equilibrium outcomes. For most of the proposed scenarios, this assumption of ex-post symmetry can be fully justified. However, for the R&D cartel, as Salant and Shaffer (1998) have shown, this restriction may well result in a failure to achieve the global maximum of the joint payoff function. (This is due to the possible failure of joint concavity of the total payoff in the two R&D decisions, in spite of the assumed concavity of each payoff in own decision.) In other words, identical firms colluding in their R&D choices and then competing in a product market may well find it advantageous to settle for different levels of R&D, thereby ending up as unequal rivals in the product market.

The present paper has two objectives. First, an attempt is made to generalize and unify the previous results of this literature. In particular, the specifications of linear demand and production costs, quadratic R&D costs, as well as the traditional separation between Cournot and Bertrand cases, are removed. This is accomplished by representing the product market competition at the second-stage by a function  $\Pi(\cdot, \cdot)$ , which gives a firm's equilibrium profit in the product market as a function of the two post-R&D unit costs. This approach requires only the most basic assumptions on the product market primitives, which are satisfied very broadly, in particular under all the common specifications of Cournot competition with differentiated or homogeneous products and Bertrand competition with differentiated products. Thus our analysis provides a unified treatment of strategic R&D encompassing most known specifications of product market competition. Throughout the present paper, an ancillary objective is to employ only minimally sufficient assumptions on  $\Pi$  needed for our results, thereby preserving as high a level of generality as possible, while also formally establishing the existence of a unique symmetric subgame-perfect equilibrium for the two-stage game at hand.<sup>3</sup>

The second main purpose of this paper is to formalize the concept of an optimal R&D cartel, from the firms' standpoint, and provide a characterization of its properties. In an optimal R&D cartel, participating firms choose the level of the spillover parameter in the interval  $[\underline{\beta}, 1]$  in addition to the (possibly asymmetric) levels of R&D for each firm (where  $\underline{\beta}$  is the lowest feasible spillover rate for the industry at hand) in order to maximize joint profits. By so endogenizing the value of the spillover rate for an R&D cooperative, one clearly captures the fullest scope of cooperative behavior possible along the two dimensions described above. It is natural to postulate that when contemplating R&D cooperation, firms would seek to find the most profitable way of doing so, provided it is legal and implementable. As a by-product of the analysis of this new issue, we further generalize the results of AJ and KMZ by dispensing with the rather disturbing assumption

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<sup>3</sup> While quite a few earlier related studies also deal with very general models (including Brander and Spencer (1983), Spence (1984), Katz (1986), Suzumura (1992), Simpson and Vonortas (1996)), they do not address the issue of existence of subgame-perfect equilibrium, or its uniqueness, or how the failure of the latter may affect relevant comparative statics results derived from the model. By contrast, we provide a minimal set of assumptions leading to a unified rigorous treatment incorporating all the issues raised here, in addition to the removal of the unjustified ex-post symmetry assumption.

of ex-post symmetry for R&D cartels, thereby addressing the criticism raised by Salant and Shaffer (1998, 1999). See also Long and Soubeyran (2001).

Our results may be summarized as follows. For the noncooperative R&D scenario, we establish the existence and uniqueness of a symmetric subgame-perfect equilibrium for the two-stage game, under our new more general set-up. Also, we show that the equilibrium effective R&D level is decreasing in the spillover parameter, which is in accord with basic economic intuition. Keeping the assumption of ex-post symmetry, we show that the principal result from previous studies on the overall superiority of the cartelized RJV over all the other scenarios essentially carries over to our framework.

The question of whether this important result remains valid without the assumption of ex-post symmetry is naturally embedded in the more general issue of optimal R&D cartel design (with endogenous spillover rate) for the firms. The main result in our general framework establishes that the optimal R&D cartel must either be the cartelized RJV (with spillover equal to 1), or else call for all the R&D to be conducted by one firm only. Furthermore, with a natural weak convexity assumption on  $\Pi$ , the latter case always involves, in addition, an optimal choice of minimal spillover (i.e.,  $\underline{\beta}$ ), a somewhat surprising general result for which we provide a natural economic interpretation. Finally, under a strong convexity condition on the R&D cost function (beyond what is needed for the analysis of noncooperative R&D), firms would always find it globally optimal to choose equal levels of R&D in a cartel, thereby leading again to the superiority of the cartelized RJV.

Furthermore, in the specific context of Cournot competition with linear demand and production costs, and quadratic R&D costs, we provide specific conditions under which the optimal cartel involves a zero spillover choice (and has only one firm conducting R&D). Since this example fits the framework of KMZ, we conclude that their main result (about the superiority of the cartelized RJV) relies crucially on their ex-post symmetry restriction or on other sufficient conditions on the primitives of the model, which we identify for the example at hand.

This paper is organized as follows. The noncooperative model is described and its basic properties derived in Section 2. Section 3 deals with cooperative R&D, comparing various R&D scenarios with an exogenously fixed spillover rate, and then considers the optimal design of an R&D cartel (from the firms' point of view). An illustrative example is presented in Section 4, followed by a brief conclusion in Section 5. Finally, all proofs are given in Section 6.

## **2. Noncooperative R&D**

### *2.1. The model*

Consider an industry composed of two identical firms, each with initial unit (production) cost  $c > 0$ , engaged in the following two-stage game. In the first stage, Firms 1 and 2 simultaneously conduct process R&D, choosing autonomous R&D expenditures  $x_1$  and  $x_2$ , respectively. In the second stage of the game, upon observing  $x_1$  and  $x_2$ , the two firms engage in product market competition, described below in rather broad terms encompassing both Cournot and Bertrand cases.

Without R&D spillovers, the cost reduction corresponding to an autonomous expenditure  $x_i$  is given by  $f(x_i)$ , where  $f : [0, \infty) \rightarrow [0, c]$ . However, R&D spillovers form a key part of this model. As in Spence (1984) and KMZ,<sup>4</sup> spillovers are formulated as follows: given autonomous R&D outlays  $x_1$  and  $x_2$ , the effective R&D outlays of Firms 1 and 2 are  $X_1 = x_1 + \beta x_2$  and  $X_2 = x_2 + \beta x_1$ , respectively, where  $\beta \in [0, 1]$  is a spillover parameter. Thus,  $\beta = 0$  means R&D is perfectly appropriable while  $\beta = 1$  means R&D is a pure public good.

Attention is restricted to subgame-perfect equilibria in pure strategies throughout the paper. Thus, a pure strategy for Firm  $i$  in the two-stage game is a pair  $(x_i, \sigma_i)$  where  $x_i \geq 0$  and  $\sigma_i : [0, \infty)^2 \rightarrow [0, \infty)$  is a map from (autonomous) R&D outlays to product market decisions (i.e., prices or quantities).

The following assumptions are in effect throughout the paper:

- (A.1) There exists a Nash equilibrium selection in the second-stage game (of product market competition), that can be identified for every pair of first-stage R&D decisions and, when the latter are equal, specifies a symmetric equilibrium.

Let  $\Pi : [0, c]^2 \rightarrow \mathbb{R}$  denote the corresponding equilibrium profit function (for the second-stage game). Here, given post-R&D unit costs  $c_1$  and  $c_2$  for the two firms,  $\Pi(c_1, c_2)$  is the equilibrium profit of the firm whose unit cost is  $c_1$  (i.e., the first argument).

- (A.2)  $\Pi : [0, c]^2 \rightarrow \mathbb{R}$  is twice continuously differentiable and satisfies:<sup>5</sup>

- (i)  $\Pi_1 < 0$  and  $\Pi_2 > 0$ .
- (ii)  $|\Pi_1(z, z)| > \Pi_2(z, z)$ ,  $\forall z \in [0, c]$ .

- (A.3)  $f : [0, \infty) \rightarrow [0, c]$  is twice continuously differentiable and satisfies:

- (i)  $f(0) = 0$ ,  $f(\infty) = c$ ,  $f' > 0$  and  $f'' < 0$ .
- (ii)  $f'(0) = \infty$  and  $f'(\infty) = 0$ .

- (A.4) (i)  $\forall \beta \in [0, 1]$ , we have, with the  $\Pi$  terms evaluated at  $[c - f(x_1 + \beta x_2), c - f(x_2 + \beta x_1)]$ :

$$-\Pi_{11}f'^2(x_1 + \beta x_2) + \Pi_1f''(x_1 + \beta x_2) - \beta\Pi_{12}f'(x_1 + \beta x_2)f'(x_2 + \beta x_1) > 0,$$

$$\forall x_1, x_2 \geq 0.$$

- (ii)  $\forall \beta \in [0, 1]$  and  $z \in [0, c]$ , we have, with  $g \doteq f^{-1}$  (the inverse function),

$$\Delta_\Pi(z) < g''(z),$$

where  $\Delta_\Pi(z) \doteq \Pi_{11}(c - z, c - z) + (1 + \beta)\Pi_{12}(c - z, c - z) + \beta\Pi_{22}(c - z, c - z)$ .

We now interpret and discuss the meaning and scope of these conditions, emphasizing in particular their relationship to their counterparts in related work. Assumption (A.1) allows for a broad scope of product market competition modes, including in particular

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<sup>4</sup> See Amir (2000) for a comparative critique of the other prevalent way to model spillovers, through cost reductions, as in AJ.

<sup>5</sup> Throughout, a subscript  $i$  will denote the partial derivative with respect to the  $i$ th variable.

Cournot and Bertrand specifications.<sup>6</sup> The equilibrium selection assumption is convenient and relatively general. A clear-cut way for (A.1) to hold is to have a unique equilibrium in the second-stage game. In the case of Cournot competition with linear costs, Amir (1996) shows that this holds whenever  $P(\cdot) - c_i$  is a log-concave function, where  $P(\cdot)$  is the inverse demand function and  $c_i$  is the unit cost of Firm  $i$ ,  $i = 1, 2$ . This is implied by  $P(\cdot)$  itself being log-concave, and is easily seen to be satisfied by most of the widely used specifications. For Bertrand competition with differentiated products, Milgrom and Roberts (1990) give a uniqueness argument with several illustrative examples.

However, (A.1) does not require uniqueness of the second-stage equilibrium, but only that a particular selection be identified for all R&D choices in the first stage. For instance, in many cases, this can take the form of a maximal or minimal equilibrium for the second stage, according to some order on the action sets. These extremal selections are often natural in that they have some justifying property, such as Pareto-dominance for the firms or the consumers: see, e.g., Milgrom and Roberts (1990) and Amir and Lambson (2000).

(A.2)(i) is self-explanatory: a firm's profits decrease with own cost, but increase with rival's cost. (A.2)(ii) says that in a symmetric duopoly, an equal decrease in both firms' costs raises their profits. Put differently, own cost effects dominate rival's cost effects on profit. These are both satisfied very broadly.

With (A.3) clearly reflecting the natural assumption of diminishing returns in R&D along with customary Inada-type conditions, (A.4) may be viewed as a strengthening of the same property, as we argue below. (Note that a version of (A.4) is standard in the literature: see, e.g., p. 1134 in AJ or pp. 1298–1299 in KMZ.) Since  $g \doteq f^{-1}$ ,  $g : [0, c] \rightarrow [0, \infty)$  can be thought of as the process-R&D cost function, in the sense that it costs  $g(z)$  for a firm to (autonomously) reduce its unit cost by  $z \in [0, c]$ . An alternative way to state (A.3)(i) is then that  $g$  is twice continuously differentiable, with  $g(0) = 0$ ,  $g(c) = \infty$ ,  $g' > 0$  and  $g'' > 0$ .

While (A.4) clearly imposes a joint restriction on  $f$  or  $g$  and  $\Pi$ , it is most instructive to view it as a condition on  $g$  for given  $\Pi$ . Indeed, since  $\Pi$  is smooth on  $[0, c]^2$ , all its partials are uniformly bounded on  $[0, c]$ . Hence (A.4) typically imposes a lower bound on the degree of concavity of  $f$  or convexity of  $g$ .<sup>7</sup> If, as is the case in previous models in this literature,  $\Pi_{12} < 0$ , it is sufficient for (A.4)(i) that the first two terms have a positive sum, which is the same as  $\Pi[c - f(x_1), c - f(x_2)]$  being concave in  $x_1$ , for fixed  $x_2$ . A version of the latter condition is standard in this literature.

As an example, provided  $g$  is sufficiently convex (in the sense that  $g''$  or  $|f''|$  is large enough), all of our assumptions here are satisfied in the most commonly used case of Cournot competition with linear demand  $P(Q) = a - bQ$  and unit costs  $k_1$  and  $k_2$ ,<sup>8</sup> and

<sup>6</sup> It is worth noting that postulating  $\Pi$  as an equilibrium payoff function for the product market competition also allows for the following interesting interpretation:  $\Pi$  represents the overall payoff to a multi-stage game (possibly with infinite-horizon) in the product market. In such a setting, for the two-stage game at hand, the R&D decision is a long-term decision followed by many short-term market decisions.

<sup>7</sup> Furthermore, the level of generality of (A.4) is enhanced by the absence of specific sign assumptions on each of the second partials of  $\Pi$ . In particular, the negative complementarity condition  $\Pi_{12} < 0$  (or submodularity of  $\Pi$ , see Topkis, 1978) has a natural interpretation here and would make it easier for both parts of (A.4) to hold, but it is not actually needed. Also, see Athey and Schmutzler (1995, 2001).

<sup>8</sup> This leads to equilibrium profits for Firm 1 (say) given by  $\Pi(k_1, k_2) = (a - 2k_1 + k_2)^2/9b$ .

AJ's quadratic cost function<sup>9</sup>  $g(z) = \gamma z^2/2$ . Then (A.4)(ii) boils down to  $\Delta\pi = 2(2 - \beta) < 9b\gamma$ . Analogous remarks apply to Bertrand competition with differentiated products, linear demand  $q_i = a - p_i + bp_j$ ,  $0 < b < 1$ ,  $i, j = 1, 2$ ,  $i \neq j$ , and units costs  $k_1, k_2$ , which leads to equilibrium profits for Firm 1 of  $\pi(k_1, k_2) = [(2+b)a - (2-b^2)k_1 + bk_2]^2$ , the details being left out.

Overall, we hope to have shown that this set of assumptions yields a rather general framework (relative to the related literature), which leaves open the possibility that the second-stage game may encompass other modes of competition (in addition to Cournot and Bertrand).

In view of assumption (A.1) and the restriction to subgame-perfect equilibria, the payoff function of Firm 1 (say), given R&D decisions  $x_1, x_2 \geq 0$ , is

$$F(x_1, x_2) = \pi[c - f(x_1 + \beta x_2), c - f(x_2 + \beta x_1)] - x_1. \quad (2.1)$$

By symmetry, Firm 2's payoff is  $F(x_2, x_1)$ . Since every Nash equilibrium of the game with payoffs (2.1) induces a subgame-perfect equilibrium of the two-stage game at hand and vice-versa, we will use the two equilibrium notions interchangeably below.

## 2.2. The results

This subsection provides the main results for our noncooperative R&D model: existence and uniqueness of a symmetric subgame-perfect equilibrium, and its comparative statics with respect to the spillover parameter  $\beta$ .

**Proposition 2.1.** *Under assumptions (A.1)–(A.4), the two-stage game has a unique symmetric subgame-perfect equilibrium, for every  $\beta \in [0, 1]$ .*

In view of Proposition 2.1, the game with payoffs (2.1) has a unique symmetric (and possibly other asymmetric) equilibrium. As will become clear in the proof of Proposition 2.1, both existence and uniqueness of a symmetric equilibrium follow from the fact that assumptions (A.1)–(A.4) lead to the reaction curve of a firm being a (single-valued) continuous function that is decreasing at any intersection with the diagonal.

Denote by  $(x^N, x^N)$  the symmetric equilibrium R&D expenditures. The following comparative statics result is of independent interest and will be invoked repeatedly in the analysis of cooperative R&D (in Section 3).

**Proposition 2.2.** *Under assumptions (A.1)–(A.4), each firm's effective equilibrium R&D level  $X^N = (1 + \beta)x^N$  is nonincreasing in  $\beta \in [0, 1]$ .*

The conclusion of Proposition 2.2 is interpretable along standard lines: as  $\beta$  increases, R&D becomes more and more of a public good, inducing each firm to spend less on autonomous R&D and thus to free ride more on its rival's R&D. Observe that the reduction of

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<sup>9</sup> The AJ cost function fails to satisfy the natural requirement that  $g(c) = \infty$ , and as a consequence, full cost reduction by both firms may well be the unique equilibrium in their model. This drawback is outweighed by the computational advantages of the quadratic form.

own R&D due to an increase in the spillover level is drastic since even *effective* R&D levels fall. In other words,  $X^N$  decreasing in  $\beta$  is a stronger statement than  $x^N$  decreasing in  $\beta$ .

### 3. R&D cooperation

Following AJ and KMZ, we consider two possible (independent) dimensions of R&D cooperation for firms contemplating participation:

- (i) whether firms coordinate their R&D decisions (i.e., expenditures) by maximizing the industry (total) profit, and
- (ii) whether firms engage in know-how sharing by mutually agreeing to internally set the final spillover parameter  $s$  at some value in  $[\underline{\beta}, 1]$ , where  $\beta$  is the original or natural spillover rate, a characteristic of the industry under consideration (reflecting technological, locational and human factors, among others).

Put differently, with the firms remaining competitors in the product market, (i) amounts to collusion in the R&D phase of the game, and (ii) models R&D information sharing as an increase in the spillover rate, with  $s = 1$  corresponding to an integrated (or joint) lab.

In the second part of this section, we depart from the bulk of the literature in that we consider the choice of the final spillover parameter  $s$  as one of the design variables for firms entering an RJV agreement. Thus, while previous papers regarded the spillover rate for an R&D cartel as being either  $\beta$  or 1, we endogenize its value here. Furthermore, we do so in the interval  $[\underline{\beta}, 1]$ , instead of just  $[\beta, 1]$ , with  $\underline{\beta} \in [0, \beta]$  representing the lowest feasible level of the spillover rate. (As will be seen and interpreted below, optimal total profits may be highest in the complete absence of spillovers.) However, while increasing  $s$  in the interval  $[\underline{\beta}, 1]$  clearly corresponds to increasing the R&D information flow across firms, choosing  $s$  in  $[\underline{\beta}, \beta]$  is not as readily interpretable, at least not within the confines of the present model. Nonetheless, depending on the type of industry at hand, one could think of this as a reflection of any subset of the following possibilities for the cartel firms:

- (i) locating further away from each other;
- (ii) entering “agreements” not to hire away each other’s scientists;
- (iii) camouflaging their products and processes more intensively than before;
- (iv) agreeing to choose more differentiated products; and/or
- (v) agreeing to choose unrelated R&D approaches or paths.<sup>10</sup> With this discussion to be continued after the results, we now describe the different forms of R&D cooperation.

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<sup>10</sup> A more elaborate model may attempt to endogenize some of these possible features, e.g., by adding an initial period to our two-stage game where firms would make the corresponding decision(s). Such an approach may shed some light on issues of substantial current interest: (a) economic geography issues such as the emergence of technology parks (e.g., Silicon Valley, Route 128, ...), and (b) the relationship between the choice of product line and process R&D. While our results below, treating  $\beta$  as a parameter reflecting a proxy measure of these decisions, would clearly partly pave the way for such an endeavor, this is beyond the scope of this paper.

The joint objective of the two firms with the final spillover rate exogenously set at  $s \in [\underline{\beta}, 1]$  is given by

$$\begin{aligned} H_s(x_1, x_2) = & \Pi[c - f(x_1 + sx_2), c - f(x_2 + sx_1)] \\ & + \Pi[c - f(x_2 + sx_1), c - f(x_1 + sx_2)] - x_1 - x_2. \end{aligned} \quad (3.1)$$

We will refer to this scenario as case  $C_s$ , where  $C$  stands for coordinated or collusive. Likewise, we refer to the noncooperative solution of Section 2 as case  $N$ .

Let case  $C^*$  stand for the optimal R&D cartel<sup>11</sup> which solves

$$\max\{H_s(x_1, x_2); s \in [\underline{\beta}, 1], x_1 \geq 0, x_2 \geq 0\}, \quad \text{with } \underline{\beta} \in [0, \beta]. \quad (3.2)$$

We now define another R&D cooperation scenario, the Joint Lab, to be referred to as case  $J$ , whereby the two firms jointly run one R&D lab at half the cost each, and thus end up with the same final cost reduction. Letting  $x$  be the total R&D expenditure of the joint lab (with  $x_1 = x_2 = x/2$ ), the joint objective of the firms is

$$\max_x \{2\Pi[c - f(x), c - f(x)] - x\}. \quad (3.3)$$

### 3.1. R&D cooperation with an exogenous $\beta$

We begin with a formal proof of the equivalence between cases  $J$  and  $C_1$ , a fact which, as suggested earlier, is rather intuitive.

**Lemma 3.1.** *Cases  $J$  and  $C_1$  are equivalent, in the sense that they lead to the same final cost reduction (for both firms) and the same total profit.*

Given this equivalence, one may wonder about the motivation for defining case  $J$ . The answer is simply that

- (i) case  $J$  is easier to conceptualize economically than case  $C_1$  (a joint lab with equal R&D cost-sharing is more concrete than R&D collusion coupled with full sharing of know-how),
- (ii) the equal cost split is directly built into case  $J$  while case  $C_1$  leaves the allocation of total R&D cost indeterminate, and
- (iii) case  $J$  is defined independently of  $\beta$ .

Some of our results require a strengthening of (A.2)(ii). Assumption (A.5) quantifies the dependence of equilibrium profits on own versus cross cost reductions in a symmetric duopoly setting.

$$(A.5) \quad |\Pi_1(z, z)| \geq 2\Pi_2(z, z), \forall z \in [0, c].$$

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<sup>11</sup> Throughout the paper, optimal R&D cartel refers to a joint-profit maximizing cartel, and not to a socially optimal cartel.

Clearly, (A.5) is a stronger requirement than (A.2)(ii). We now argue that (A.5) is not as restrictive as it might appear at first. It is satisfied under Cournot competition with linear demand and costs (with strict inequality if products are differentiated and with equality for homogeneous products, see Section 2 for the expression of  $\Pi$ ). For Bertrand competition with differentiated products, (A.5) can be seen to hold if and only if the cross-demand coefficient (denoted by  $b$  in the discussion of (A.1)–(A.3) in Section 2) is in the interval  $(0, \sqrt{3} - 1] \approx (0, 0.73]$ ,<sup>12</sup> i.e., as long as demand is different enough from the well-known case of homogenous products ( $b = 1$ ).

The first result deals with the comparison of R&D propensities between the two cases of primary interest:  $N$  versus  $J$  (a detailed discussion of all the results is given following their statement).

**Proposition 3.2.** *Under assumptions (A.1)–(A.5),  $x^J \geq X^N$ , for all  $\beta \in [0, 1]$ .*

Both AJ and KMZ imposed a symmetry restriction on case  $C_\beta$ . In other words, they assumed the two firms jointly maximize (3.1) with  $s = \beta$  (the original exogenous spillover rate), subject to the constraint  $x_1 = x_2$ . In an insightful note, Salant and Shaffer (1998) pointed out that the globally optimal solution for case  $C_\beta$  may well fail to be symmetric under the assumptions of AJ and KMZ. This is due to the fact that concavity of each firm's payoff in own decision need not imply joint concavity of  $H_\beta(x_1, x_2)$  in  $x_1$  and  $x_2$ : see (3.1). (Note that with joint concavity of  $H_\beta$  in  $(x_1, x_2)$ , the optimal solution would always involve equal choices of R&D outlays  $x_1$  and  $x_2$  by the two firms, due to the fact that  $H_\beta$  is symmetric in  $x_1$  and  $x_2$ : see Proposition 3.9 below.)

It turns out that the symmetric version of  $C_\beta$ , to be denoted case  $\bar{C}_\beta$ , is useful below when comparing optimal total profits under different scenarios. Let  $H_s^*$  denote the optimal total profit that can be obtained in case  $C_s$  (i.e.,  $H_s^*$  is the maximum value in (3.1)). Likewise, let  $\bar{H}_s^*$  denote the optimal total profit that can be obtained in case  $C_s$  with the additional constraint of a symmetric choice  $x_1^* = x_2^*$ . Finally, let  $H_N^*$  and  $H_J^*$  be total equilibrium profit in cases  $N$  and  $J$ .

The following result generalizes KMZ's profit comparison result.

**Proposition 3.3.** *Under assumptions (A.1)–(A.4), the optimal total profits for cases  $N$ ,  $\bar{C}_\beta$ ,  $C_\beta$ , and  $J$  satisfy  $H_N^* \leq \bar{H}_\beta^* \leq \min\{H_\beta^*, H_J^*\}$ .*

As will be seen in the last section, the proof of Proposition 3.3 is based on the following fact (which is of independent interest) concerning the constrained symmetric case.

**Lemma 3.4.** *Under assumptions (A.1)–(A.4),  $\bar{H}_s^*$  is strictly increasing in  $s \in [0, 1]$ .*

We now turn to a welfare comparison between cases  $N$  and  $J$ . Due to the absence of the standard market primitives (such as consumer utility functions or market demand) in the

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<sup>12</sup> In their treatment of Bertrand competition, KMZ give 2/3 as a lower bound for this critical value of  $b$ . Since our model is more general, the fact that our bound is sharper indicates that (A.5) is tight (see also the proof of Proposition 3.2 for more insight on the role of (A.5)).

description of our model, a precise notion of consumer surplus cannot be defined. Instead, we make the following natural assumption.

- (A.6) Consumer surplus is nonincreasing in the firms' unit production costs. Furthermore, social surplus is the sum of consumer and producer surpluses.

This assumption holds in most commonly used specifications of Cournot and Bertrand competition. In particular, it holds for the cases of linear demand reported in Section 2. For Cournot competition (with homogeneous products), it actually holds for any demand function, provided production costs are linear and a Cournot equilibrium exists. This is because total output at equilibrium (and hence price) depends only on total unit cost (Bergstrom and Varian, 1985).

A direct consequence of Propositions 3.2 and 3.3 is:

**Corollary 3.5.** *Under assumptions (A.1)–(A.6), social surplus is higher under case J than under case N.*

So far, we have confined our analysis of R&D cooperation to cases where the firms always cartelize their R&D decisions in conjunction with various levels of R&D information sharing (i.e., setting the value of the spillover rate  $s$ ). Now, following KMZ's case NJ, we consider a scenario where firms behave noncooperatively in their R&D decisions (in addition to their product market decisions as before), but enhance their R&D information sharing by setting  $s$  equal to one. Then each firm's R&D expenditure is  $x^{NJ} = x^N|_{\beta=1}$ . We show that this scenario lowers R&D activity and is thus harmful to consumer welfare.

**Proposition 3.6.** *Under assumptions (A.1)–(A.4), and (A.6), we have  $X^{NJ} \leq X^N$  and thus lower consumer welfare in case NJ than in case N.*

We now relate the results of this subsection to KMZ and Salant and Shaffer (1998). As mentioned earlier, the main purposes of the present paper are to

- (i) extend the main results of KMZ to a more general framework, thereby identifying the critical features of the two-stage duopoly that drive the conclusions,
- (ii) incorporate into the analysis the critique of Salant and Shaffer by removing the imposed constraint of symmetric R&D choices for R&D cartels, and
- (iii) endogenize the value of the spillover parameter  $s$  in designing an optimal R&D cartel.  
((ii) and (iii) form the subject of the next subsection and are discussed there.)

Propositions 3.2 and 3.3 and Corollary 3.5 form a generalization of the main result of KMZ, which says that a joint lab yields a superior performance compared to noncooperative R&D in all three criteria of interest: propensity for R&D, firms' profit (or producer surplus), and consumer surplus, and thus also social welfare. Since KMZ assumes linear demand with differentiated products, the generalization presented here is a useful robustness check for the important policy conclusions drawn from these models.

The antitrust implications of our theoretical conclusions are unambiguous: simultaneous R&D sharing and coordination of R&D decisions among firms that compete in a product market should be permitted and encouraged, at least for industries in which firm size is more or less homogeneous. In view of Proposition 3.3 though, the firms' incentives for cooperation in R&D are clear, and no direct government action should be necessary, other than providing the attending legal framework for such cooperative arrangements. As with the rest of the related literature, the possibility that cooperation in R&D may pave the way for, or at least increase the likelihood of, collusive behavior in the product market is not considered in the present paper. This important point is raised and discussed by Martin (1995) in a repeated-games framework.

Proposition 3.6 generalizes an auxiliary result of KMZ that nonetheless has interesting and subtle antitrust implications. It maintains that cooperation in the conduct of R&D (or R&D information-sharing) that is not simultaneously accompanied by the coordination of R&D decisions (here expenditures) is detrimental to consumer welfare. Due to its anticompetitive nature, this partial form of cooperation should not be permitted by antitrust authorities.

### 3.2. The optimal R&D cartel (case C\*)

Here, we investigate the optimal choice of the spillover rate by firms forming an R&D cartel, i.e., we solve the optimization problem described in (3.2). (Note that in (3.2) there is no symmetry restriction on the R&D outlays.) Our main result in this subsection essentially says that firms in an RJV would always settle for one of two possible choices when the spillover rate is endogenous: either  $s^* = 1$  and  $x_1^* = x_2^* = x^J/2$  (so that the joint lab is optimal), or else  $s^* < 1$  and then either  $x_1^*$  or  $x_2^*$  must be 0 (so that one firm performs no R&D). With an additional plausible assumption on the dependence of equilibrium profits on unit costs, the latter case actually has  $s^* = \underline{\beta}$ . In an example with Cournot competition, linear demand, and quadratic R&D costs, we fully illustrate these surprisingly general results. We begin with an intermediate result that captures the role of interiority of R&D decisions and is central to the main proposition below.

**Lemma 3.7.** *In addition to assumptions (A.1)–(A.4), let  $\underline{\beta} \leq a < b \leq 1$ , and suppose that for all  $s \in [a, b]$ , (at least) one  $\arg\max(x_1^*, x_2^*)$  in (3.1) is interior, i.e.,  $x_1^* > 0$  and  $x_2^* > 0$ . Then the optimal total profit  $H_s^*$  is strictly increasing in  $s \in [a, b]$ .*

Lemma 3.7 essentially constitutes a generalization of Lemma 3.4 where the same conclusion is obtained under the imposed restriction that the firms must invest equal R&D outlays. Clearly, in Lemma 3.7, the optimal choice need not satisfy  $x_1^* = x_2^*$ . Considering the level of generality of the required assumptions, the message from this result is rather interesting. It essentially says that as long as the optimal R&D outlays are both strictly positive, the same total profit can be obtained with lower outlays provided the spillover rate is appropriately increased.

The main result of this subsection is:

**Proposition 3.8.** (a) Under assumptions (A.1)–(A.4), the optimal R&D cartel in (3.2) must satisfy either

- (i)  $s^* = 1$  and  $x_1^* + x_2^* = x^J$ , or
- (ii)  $s^* < 1$  and  $\min\{x_1^*, x_2^*\} = 0$ .

(b) If, in addition to (A.1)–(A.4),  $\Pi$  satisfies

$$\Pi_{11}(c_1, c_2) + \Pi_{22}(c_2, c_1) > 0, \quad \text{for all } c_2 > c_1 \text{ in } [0, c] \quad (3.4)$$

then the optimal cartel satisfies either

- (i)  $s^* = 1$  and  $x_1^* + x_2^* = x^J$ , or
- (ii)  $s^* = \underline{\beta}$  and  $\min\{x_1^*, x_2^*\} = 0$ .

Assumption (3.4) is natural and satisfied in all the standard specifications. In particular, for both Cournot and Bertrand competition with linear demand and differentiated products,  $\Pi$  is actually jointly convex in  $(c_1, c_2)$ , a much stronger property than (3.4). Another example is Cournot competition with quadratic demand  $P(Q) = a - bQ^2$ ,  $Q \leq \sqrt{a/b}$ , leading to equilibrium profit equal to  $\Pi(k_1, k_2) = \frac{1}{16}(2a + k_2 - 3k_1)^2/\sqrt{b(2a - k_1 - k_2)}$ , which satisfies  $\Pi_{11} > 0$  and  $\Pi_{22} > 0$  for all  $(k_1, k_2)$ , again implying (3.4).

Thus the optimal design of an R&D cartel leads to two possible outcomes: either the firms find it jointly profitable for both to conduct R&D, in which case a joint lab (i.e., full communication of R&D know-how, cf. Lemma 3.1) is the optimal R&D cartel, or only one firm engages in R&D, leading to unequal market shares in the product market, with the rival firm improving its unit cost only through spillovers. Under (3.4), the result becomes much sharper in that only extremal choices of the spillover rate are possible, a rather provocative result. An economic interpretation of this result is given in the next section in the context of a commonly chosen example wherein a fuller characterization is possible. Furthermore, some real-world and empirical aspects of these suggestive results, as well as some possible empirical tests, are presented in light of the more explicit findings derived there.

We now return to the discussion of the plausibility of lowering  $\beta$ . Even in contexts in which none of the four possibilities given at the beginning of this section is plausible, detailed insight into the role of the spillover rate in the design of the best R&D cartel for the firms remains desirable for a complete understanding of the incentives at work in an R&D cartel. In particular, we now observe that under (3.4), if  $\beta$  cannot be lowered at all, then case  $C_\beta$  is preferable to the firms than anything other than case  $C_1$ .

**Corollary 3.9.** In addition to assumptions (A.1)–(A.4), and (3.4), suppose that  $\underline{\beta} = \beta$ . Then the optimal R&D cartel is either case  $J$  or case  $C_\beta$ .

Our last result on the structure of optimal R&D cartels shows that if the returns to R&D are sufficiently decreasing (i.e.,  $f$  is concave enough or  $g \doteq f^{-1}$  is convex enough), the firms would always find it optimal to choose equal R&D investments, as the joint objective function (3.1) will then be jointly concave in  $(x_1, x_2)$ , in addition to being symmetric in  $(x_1, x_2)$ .

**Proposition 3.10.** *In addition to assumptions (A.1)–(A.4), assume that for all  $a \in [0, c]$*

$$(1+s)^{-1}g''(\cdot) \\ > \Pi_{11}(c - (\cdot), a) + \Pi_{22}(a, c - (\cdot)) + |\Pi_{12}(c - (\cdot), a) + \Pi_{21}(a, c - (\cdot))|. \quad (3.5)$$

*Then, for each  $s \in [0, 1]$ , the optimal R&D choices are equal and the corresponding optimal total profit  $H_s^*$  is strictly increasing in  $s$ . In particular, the joint lab is the optimal R&D cartel.*

#### 4. Optimal R&D cartel with Cournot competition

Here we consider an example in which at the first stage the R&D cost function is given by  $g(z_i) = \gamma z_i^2/2$  where  $z_i \in [0, c]$  is Firm  $i$ 's autonomous cost reduction (as in AJ, or equivalently  $f(x_i) = \sqrt{2x_i/\gamma}$ ) and at the second (product market competition) stage, the firms are Cournot competitors with demand in the output market given by  $P(q_1, q_2) = a - (q_1 + q_2)$ . We assume that  $a > 2c$  which insures that every subgame at the second stage has a unique Nash equilibrium, with both firms in the market. In the subgame in which firms  $i$  and  $j$ 's post-R&D unit costs are  $c_i$  and  $c_j$ , respectively, Firm  $i$ 's Nash output and profit are, respectively,  $(a - 2c_i + c_j)/3$  and  $\Pi(c_i, c_j) = (a - 2c_i + c_j)^2/9$ . Firm  $i$ 's profit in the overall game given R&D expenditures  $x_i$  and  $x_j$  is, therefore,

$$F(x_i, x_j) = \left( a - c + 2\sqrt{\frac{2}{\gamma}(x_i + \beta x_j)} - \sqrt{\frac{2}{\gamma}(x_j + \beta x_i)} \right)^2 / 9 - x_i.$$

R&D expenditures and effective R&D expenditures in cases  $N$  and  $J$  are given in the following table.<sup>13</sup>

	R&D expenditure	Effective R&D expenditure
Case $N$	$x^N = \frac{2\gamma(a-c)^2(2-\beta)^2}{(1+\beta)(9\gamma+2\beta-4)^2}$	$X^N = \frac{2\gamma(a-c)^2(2-\beta)^2}{(9\gamma+2\beta-4)^2}$
Case $J$	$x^J = \frac{8\gamma(a-c)^2}{(9\gamma-4)^2}$	$X^J = x^J$

Simple calculations establish that  $x^J \geq x^N$  and  $X^J \geq X^N$ , with both inequalities strict for  $\beta > 0$ .

The joint objective of the two firms with the spillover rate  $s$  is

$$H_s(x_1, x_2) = \left( a - c + 2\sqrt{\frac{2}{\gamma}(x_1 + sx_2)} - \sqrt{\frac{2}{\gamma}(x_2 + sx_1)} \right)^2 / 9 \\ + \left( a - c + 2\sqrt{\frac{2}{\gamma}(x_2 + sx_1)} - \sqrt{\frac{2}{\gamma}(x_1 + sx_2)} \right)^2 / 9 \\ - x_1 - x_2. \quad (4.1)$$

<sup>13</sup> We omit these calculations, as well as the calculations showing the concavity of each firm's profit in own decision. For the table, we assume that  $9\gamma \geq 4a/c$ , which insures that both effective R&D levels are interior (i.e.,  $X^N < c$  and  $X^J < c$ ).

The main result of this section has three parts. Part (i) shows that the optimal R&D cartel has either no or full spillovers. Part (ii) shows that if R&D costs are sufficiently convex or demand is high relative to initial unit costs, then the optimal R&D cartel is the joint lab. Part (iii) shows that if R&D costs are not too convex and demand is low relative to initial unit costs, then the optimal R&D cartel has no spillovers with one firm reducing its cost by  $c$  (with an expenditure of  $\frac{\gamma}{2}c^2$ ) and the other firm conducting no R&D.

**Proposition 4.1.** *For the particular specification at hand, let  $(s^*, x_1^*, x_2^*)$  be an optimal R&D cartel. Then:*

- (i)  $s^* = 0$  or  $s^* = 1$ .
- (ii) If either  $9\gamma > 18$  or  $\frac{a}{c} > \frac{5}{2}$  then  $s^* = 1$  and  $(x_1^*, x_2^*) \in \{(x^J, 0), (0, x^J)\}$ .
- (iii) If  $9\gamma < 10$  and  $\frac{a}{c} < \frac{5}{2}$  then  $s^* = 0$  and  $(x_1^*, x_2^*) \in \{(\frac{\gamma}{2}c^2, 0), (0, \frac{\gamma}{2}c^2)\}$ .

The intuition behind this proposition is best described in terms of a tension between two conflicting effects. The first is an efficiency effect, that identical Cournot rivals' profits increase as the common unit cost declines (assumption (A.2)(ii)), thus pushing for the choice  $s^* = 1$ . The second effect is the joint desire for cost asymmetry: total Cournot profits are convex in the unit costs, so that the cartel payoff may be highest under maximal cost differentiation between the two firms when R&D costs are not too convex. A maximal cost gap is best-achieved under a no-spillover regime. Under this perspective, the Proposition simply identifies specific conditions for each of the two effects to be dominant: see Amir and Wooders (1999, 2000) for a similar analysis in a different model.

In words, Proposition 4.1 says that we should expect joint labs—or extensive cooperation in the conduct of R&D—when the cost of R&D is high (or  $\gamma$  is large) or when profitability (taken as a proxy for demand being large relative to cost) is relatively high. When both of these conditions fail, R&D cooperation would not involve joint R&D operations, but would rather be limited to cartelization, or coordination of R&D decisions. (There is also an intermediate range of R&D costs where the result is not conclusive as to whether full or no spillover is best for the firms.)

While thought-provoking, neither of these two extreme outcomes—a fully joint lab or cartelization of R&D decisions without any joint conduct of R&D—is actually observable in reality. One might conceivably take elaborate arrangements such as SEMATECH to be tantamount to a joint lab and minor, limited-scope RJs, to be reflections of case  $C_\beta$ . In the latter case, the main purpose of the joint venture may well be its tacit coordinating role in communicating the levels of private R&D investments to all participants, and as such would function much like a trade association.

With this identification, our results could presumably be empirically tested. There are several empirical studies of the variability of profitability across industries, so that relevant estimates are readily available in comparative form (Mueller, 1990). Likewise, appropriate measures of R&D costs may readily be found, e.g., as a percentage of total production cost or as the size of R&D personnel. R&D levels are known to be very high for instance in the chemical sector (in particular for pharmaceutical products) and in many high-tech industries such as software, and relatively low in some old-economy manufacturing sectors such as paper, textiles, mining, .... The relevant questions would obviously be whether

high profitability and/or R&D costs correlates well with the depth or scope of the observed RJs, using the CORE database or one of its elaborations (Link, 1996, and Roller et al., 1997).

It is widely accepted (Griliches, 1990) that the level of spillovers is an important determinant of the levels of noncooperative R&D in a given industry<sup>14</sup> (see Proposition 2.2). It would thus also be of interest to test whether the natural spillover level is an important determinant of whether RJV formation takes place, R&D cooperation being a way of internalizing the spillover externality. A related and more ambitious test, directly relating to the results of Propositions 3.8 and 4.1, is to investigate empirically the relationship between the spillover level and the extent of RJV involvement across industries.

## 5. Conclusion

This paper has two separate aims. First, known results on the comparative performance (in terms of R&D levels, profits and welfare) of various R&D cooperation scenarios and noncooperative R&D for firms competing in their product market are generalized in two important aspects

- (i) a framework is developed that does not rely on specific functional forms, yet is fully rigorous in its account of existence, uniqueness and comparative statics of equilibrium, and
- (ii) the implicit assumption in previous work of ex-post symmetry, which is not justified for R&D cartels, is removed.

The latter point naturally leads to the second and principal aim of the paper which is to allow for R&D cartel participants to also choose the optimal level of spillover  $s$  between a minimal (industry-specific) feasible value  $\beta$  and 1. It is shown that the optimal cartel essentially always sets  $s^* = 1$  or  $s^* = \beta$ , and, in the latter case, has all the R&D being conducted by one firm only. An economic interpretation is provided for this rather provocative result. Conditions are then given leading to  $s^* = 1$ . In the setting of the standard specification—linear demand for homogeneous goods, linear production costs and quadratic R&D costs—we provide respective sufficient conditions leading to each of the two possible outcomes. Possible empirical extensions are discussed.

## 6. Proofs

**Proof of Proposition 2.1.** We first establish the existence of a symmetric (pure-strategy) Nash equilibrium for the game with payoffs given by (2.1). It is convenient to separate the analysis into two distinct cases.

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<sup>14</sup> It is well known that natural spillover levels vary drastically across industries, and are often inversely related to the level of patent protection. High spillovers or low effective patent protection are prevalent in low-tech mature industries (e.g., paper) and low spillovers or high effective patent protection are prevalent in many R&D-intensive industries such as pharmaceutical drugs, software, etc., (Griliches, 1990).

**Case 1.**  $\beta = 0$ . Then it is easy to see that assumption (A.4)(i) simply says that Firm 1's payoff  $\Pi[c - f(x_1), c - f(x_2)]$  is strictly concave in  $x_1$  for each fixed  $x_2$ . Then each firm's reaction curve is a single-valued continuous function. Furthermore, from assumption (A.3) and the boundedness of profits, it is easy to see that there exists  $K > 0$  such that any R&D expenditure larger than  $K$  is a dominated strategy. So we may consider the effective strategy set as being  $[0, K]$  for both firms. Hence, existence of a Nash equilibrium follows from Brouwer's fixed-point theorem in a standard way.

**Case 2.**  $\beta > 0$ . With the change of variable  $y = \beta x_1 + x_2$ , Firm 1's payoff can be rewritten as, with  $x_2 \geq 0$  and  $y \in [x_2, \infty)$ ,

$$H(y, x_2) \triangleq \Pi[c - f\{\beta^{-1}(y - (1 - \beta^2)x_2)\}, c - f(y)] - \beta^{-1}(y - x_2).$$

It is easy to verify via direct differentiation that  $\partial^2 H(y, x_2)/\partial y \partial x_2 > 0$  as a result of assumption (A.4)(i). Thus  $H(y, x_2)$  has strictly increasing differences, or is strictly supermodular, in  $(y, x_2)$ . The feasible set  $[x_2, x_2 + K]$  is clearly ascending in  $x_2$ . Hence, from Topkis's Theorem (Topkis, 1978), every selection of  $y^*(x_2) \triangleq \arg \max\{H(y, x_2): y \in [x_2, \infty)\}$  is nondecreasing in  $x_2$ . This is equivalent to saying that the slopes of (every selection of)  $x_1^*(x_2)$  are larger than  $-\beta^{-1}$  since  $x_1^*(x_2) = \beta^{-1}[y^*(x_2) - x_2]$ , where  $x_1^*(x_2) \triangleq \arg \max\{F(x_1, x_2): x_1 \geq 0\}$ .

Applying Tarski's intersection point theorem<sup>15</sup> to the correspondence  $x_1^*$  from  $[0, K]$  to  $[0, K]$ , we conclude that it has a fixed point, which is clearly a symmetric Nash equilibrium of the original R&D game.

We now show uniqueness of this symmetric equilibrium,  $(x^N, x^N)$ . Since  $(0, 0)$  is not an equilibrium (as a consequence of assumption (A.3), 0 is not a best response to any expenditure level), all symmetric equilibria must be interior, and satisfy the following first-order condition (with  $X^N = (1 + \beta)x^N$ ):

$$-(\Pi_1 + \beta\Pi_2)[c - f(X^N), c - f(X^N)]f'(X^N) = 1. \quad (6.1)$$

To prove uniqueness, we show that the function

$$L(x) \doteq -(\Pi_1 + \beta\Pi_2)[c - f(x), c - f(x)]f'(x)$$

from the LHS of (6.1) is strictly decreasing in  $x$  at  $x = X^N$ . To this end, differentiate  $L(x)$  with respect to  $x$  to get:

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<sup>15</sup> As Tarski's (1955, p. 290) statement of this theorem is purely order-theoretic, we state Milgrom and Roberts' (1994) version in the reals instead: any function  $F: [0, K] \rightarrow [0, K]$  satisfying

$$\limsup_{x \uparrow x_0} F(x) \leq F(x_0) \leq \liminf_{x \downarrow x_0} F(x)$$

for all  $x_0 \in [0, K]$ , has a fixed-point. In words, a function that does not have any downward jumps must have a fixed point, even if it has upward jumps or is not monotonic. In the present proof, the fact that the slopes (of every selection of)  $x_1^*(x_2)$  are all larger than  $\beta^{-1}$  is clearly a sufficient condition for  $x_1^*(x_2)$  not to have any downward jumps.

McManus (1962) and Roberts and Sonnenschein (1976) developed earlier proofs of a special case of this theorem geared to a Cournot oligopoly application. Also, see Amir (1996) for an alternative proof in the Cournot case, using Topkis's work.

$$\begin{aligned}\partial L/\partial x = f'^2(x)[\Pi_{11} + (1+\beta)\Pi_{12} + \beta\Pi_{22}][c - f(x), c - f(x)] \\ - (\Pi_1 + \beta\Pi_2)[c - f(x), c - f(x)]f''(x).\end{aligned}$$

Evaluating along the first-order solution, i.e., plugging (6.1) in, yields

$$\begin{aligned}f'^2(X^N)[\Pi_{11} + (1+\beta)\Pi_{12} + \beta\Pi_{22}][c - f(X^N), c - f(X^N)] \\ + f''(X^N)/f'(X^N).\end{aligned}\tag{6.2}$$

Since  $z = f(X^N)$  if and only if  $X^N = g(z)$ , it is easily verified (by differentiating the identity  $z = f(g(z))$  twice) that  $f'(X^N) = 1/g'(z)$  and  $g''(z) = -f''(X^N)/f'^3(X^N)$ . Substituting these relations into (6.2) yields (for  $0 \leq z \leq c$ ):

$$\frac{1}{g'^2(z)}[\Pi_{11} + (1+\beta)\Pi_{12} + \beta\Pi_{22}](c - z, c - z) - \frac{g''(z)}{g'^2(z)},\tag{6.3}$$

which is negative, by assumption (A.4)(ii). Hence the LHS of (6.1) is strictly decreasing at  $X^N$ . Together with the facts that the function  $L(x)$  is continuous in  $x \in [0, \infty)$ , is infinite when  $x = 0$  and zero when  $x = \infty$  (assumption (A.3)), this implies that there is a unique solution to (6.1), so that there is a unique and interior symmetric equilibrium.  $\square$

**Proof of Proposition 2.2.** With  $X^N = (1+\beta)x^N$  we show that  $dX^N/d\beta < 0$ . Differentiating (6.1) with respect to  $\beta$  and collecting terms gives (with the same arguments as in (6.1), omitted for clarity)

$$\frac{dX^N}{d\beta} \left\{ \Pi_{11}f'^2 + (1+\beta)\Pi_{12}f'^2 + \beta\Pi_{22}f'^2 - \Pi_1f'' - \beta\Pi_2f'' \right\} = \Pi_2f'.$$

Substituting (6.1) in yields

$$\frac{dX^N}{d\beta} \left\{ f'^2[\Pi_{11} + (1+\beta)\Pi_{12} + \beta\Pi_{22}] + \frac{f''}{f'} \right\} = \Pi_2f'.\tag{6.4}$$

Since the term on the right-hand side of (6.4) is  $> 0$  (from assumptions (A.2)(ii) and (A.3)(i)), it suffices to show that the bracketed term is  $\leq 0$ . But this was already done in the proof of Theorem 2.1 (see (6.2) and (6.3)). The conclusion then follows from (6.4).  $\square$

**Proof of Lemma 3.1.** The joint objective of the firms in case  $C_1$  is obtained by setting  $\beta = 1$  in (3.1), and is

$$H_1(x_1, x_2) = 2\Pi[c - f(x_1 + x_2), c - f(x_1 + x_2)] - (x_1 + x_2).\tag{6.5}$$

Since this only depends on  $x_1 + x_2$ , setting  $x \doteq x_1 + x_2$  in (6.5), it follows that (6.1) is the same as the objective for case  $J$ , given by (3.3). It is then immediate that both firms end up with the same final cost reduction in cases  $C_1$  and  $J$ , and that total profit is also the same.  $\square$

**Proof of Proposition 3.2.** Let  $x_0$  denote the equilibrium (per-firm) R&D expenditure in case  $N$  when  $\beta = 0$  (i.e.,  $x_0 \doteq x_N$  when  $\beta = 0$ ). From Proposition 2.2, we know that  $x_0 = \max\{x_N: \beta \in [0, 1]\}$ . Hence, to establish Proposition 3.2, it suffices to show that  $x^J \geq x_0$ .

Since  $x^J$  and  $x_0$  are both interior (i.e.,  $> 0$ ) as a result of assumption (A.3)(ii), they satisfy the following first-order conditions:

$$-2(\Pi_1 + \Pi_2)[c - f(x^J), c - f(x^J)]f'(x^J) = 1 \quad (6.6)$$

and

$$-\Pi_1[c - f(x_0), c - f(x_0)]f'(x_0) = 1. \quad (6.7)$$

From assumption (A.5), it follows that

$$(\Pi_1 + 2\Pi_2)[c - f(x^J), c - f(x^J)]f'(x^J) \leq 0. \quad (6.8)$$

Adding up (6.6) and (6.8), then using (6.7) yields

$$-\Pi_1[c - f(x^J), c - f(x^J)]f'(x^J) \leq 1 = -\Pi_1[c - f(x_0), c - f(x_0)]f'(x_0).$$

From the proof of Proposition 2.1 (with the special case  $\beta = 0$ ), we know that the equation  $-\Pi_1[c - f(y), c - f(y)]f'(y) = 1$  has a unique solution (which is thus  $x_0$ ), and that for  $y \leq x_0$ , the LHS of this equation is  $\geq 1$  while for  $y \geq x_0$ , the LHS is  $\leq 1$ . Hence it follows from the above inequality that  $x^J \geq x_0$ .  $\square$

Since Lemma 3.4 implies part of Proposition 3.3, we first provide a proof of the former.

**Proof of Lemma 3.4.** In view of the symmetry constraint here, letting  $x_1 = x_2 \doteq x$  in (3.1) yields

$$H_s(x, x) = 2\Pi[c - f((1+s)x), c - f((1+s)x)] - 2x, \quad x \geq 0. \quad (6.9)$$

By assumption (A.2)(ii), this objective is strictly increasing in  $s \in [0, 1]$ . Hence, the maximum value of (6.9),  $\bar{H}_s^*$ , is strictly increasing in  $s$ .  $\square$

**Proof of Proposition 3.3.** We first show  $H_N^* \leq \bar{H}_\beta^*$ . Recall that  $H_N^*$  is the total equilibrium payoff corresponding to the (unique) symmetric equilibrium  $(x^N, x^N)$ . Since  $\bar{H}_\beta^*$  is the maximal total payoff that can be achieved via a symmetric choice  $(x, x)$ , and since  $(x^N, x^N)$  is one such feasible choice, we clearly have  $H_N^* \leq \bar{H}_\beta^*$  (note that  $s = \beta$  in both cases).

That  $\bar{H}_\beta^* \leq H_\beta^*$  follows directly from the fact that cases  $C_\beta$  and  $\bar{C}_\beta$  have the same objective function, but  $\bar{H}_\beta^*$  reflects the additional symmetry constraint  $x_1 = x_2$ .

It remains only to show that  $\bar{H}_\beta^* \leq H_J^*$ . From Lemma 3.1, we know that  $H_J^* = H_1$ , and from Lemma 3.4, we know that  $\bar{H}_1^*$  is the highest value of  $\bar{H}_s^*$  for  $s \in [0, 1]$ . Furthermore,  $\bar{H}_1^* = H_1^*$  since, with  $s = 1$ , (3.1) only depends on  $x_1 + x_2$ , and not on  $x_1$  or  $x_2$  separately. Hence,  $\bar{H}_\beta^* \leq H_J^*$ .  $\square$

**Proof of Corollary 3.5.** From Proposition 3.2, we know that  $x^J \geq x^N$  for all  $\beta$ . By assumption (A.6), it follows that consumer surplus is higher in case  $J$  than in case  $N$ . Proposition 3.3 says that producer surplus is higher in case  $J$  than in case  $N$ . Combining the two effects completes the proof of Corollary 3.5.  $\square$

**Proof of Proposition 3.6.** By definition of case  $NJ$ ,  $X^{NJ}$  is equal to  $X^N|_{\beta=1}$ . Hence, the fact  $X^{NJ} \leq X^N$  follows immediately from Proposition 2.2, and the welfare part is a direct result of assumption (A.6).  $\square$

**Proof of Lemma 3.7.** The key to this argument is a judicious change of variables in the joint maximization of the total profit function  $H_s(x_1, x_2)$ . Instead of  $(x_1, x_2)$ , one may regard the firms as choosing effective or final R&D expenditures  $(z_1, z_2)$ , such that

$$\begin{cases} z_1 \doteq x_1 + sx_2, \\ z_2 \doteq x_2 + sx_1, \end{cases} \text{ or, provided } s \neq 1, \quad \begin{cases} x_1 = \frac{z_1 - sz_2}{1 - s^2}, \\ x_2 = \frac{z_2 - sz_1}{1 - s^2}. \end{cases} \quad (6.10)$$

Provided  $s \neq 1$ , the original optimization problem for case  $C_s$ ,

$$\max \{H_s(x_1, x_2) : x_1 \geq 0, x_2 \geq 0\},$$

is equivalent to the transformed problem (via the bijection (6.10)):

$$\max \left\{ \Pi[c - f(z_1), c - f(z_2)] + \Pi[c - f(z_2), c - f(z_1)] - \frac{z_1 + z_2}{1 + s} : (z_1, z_2) \in \Delta_s \right\}, \quad (6.11)$$

where  $\Delta_s = \{(z_1, z_2) : z_1 \geq sz_2 \text{ and } z_2 \geq sz_1\} \subset [0, \infty)^2$ . Hence  $H_s^*$  is equal to the value of (6.11).

The maximand in (6.11) is strictly increasing in  $s$  while the constraint set  $\Delta_s$  is contracting in  $s$  (i.e.,  $s_1 \geq s_2 \Rightarrow \Delta_{s_1} \subset \Delta_{s_2}$ ). In view of these two conflicting effects, it cannot be concluded in general that  $H_s^*$  is increasing in  $s$ .

Nonetheless, with the additional assumption of an interior argmax on  $[a, b]$ , we now show that  $H_s^*$  is strictly increasing in  $s \in [a, b]$ . To this end, fix  $s_1 \in [a, b]$  and let  $(z_1^*, z_2^*)$  be an interior argmax of (6.11) when  $s = s_1$ . From the interiority of the argmax, we conclude that  $(z_1^*, z_2^*) \in \Delta_{s_1+\epsilon}$ , i.e.,  $(z_1^*, z_2^*)$  continues to be feasible when  $s_1$  is replaced by  $s_1 + \epsilon$  provided  $\epsilon$  is sufficiently small. Since the maximand in (6.11) is strictly increasing in  $s$ , it follows that  $H_{s_1+\epsilon}^* > H_{s_1}^*$ , for all small enough  $\epsilon > 0$ . Since  $\Delta_s$  is contracting in  $s$ , it is obvious that  $(z_1^*, z_2^*)$  is feasible when  $s_1$  is replaced by  $s_1 - \epsilon$ , thus leading (for the same reasons as above) to  $H_{s_1-\epsilon}^* < H_{s_1}^*$ . We have so far shown that  $H$  is increasing in a neighborhood of  $s_1$ . Since the choice of  $s_1$  in  $[a, b]$  is arbitrary, we conclude that  $H$  is increasing over all of  $[a, b]$ .  $\square$

**Proof of Proposition 3.8.** (a) We know from the proof of Lemma 3.1 that if  $s^* = 1$ , the joint objective  $H_1$  only depends on  $(x_1 + x_2)$ , and that then  $x_1^* + x_2^* = x^J$ . According to Lemma 3.7, this happens in particular if  $(x_1^*, x_2^*)$  is interior for all  $s \in [\underline{\beta}, 1]$ . Lemma 3.7 also implies that if  $s^* < 1$ , then  $\arg \max H_{s^*}$  cannot be interior (otherwise, taking  $s > s^*$ , but sufficiently close to  $s^*$  would be a better choice). So this argmax either has  $z_1 = s^*z_2$  or  $z_2 = s^*z_1$ , or in the original variables,  $x_1^* = 0$  or  $x_2^* = 0$ .

(b) Since (i) is the same as in part (a), we prove (ii). We know from part (a) that  $x_1^* = 0$  or  $x_2^* = 0$ . Assume (say) that  $x_2^* = 0$ . If  $s^* \neq \underline{\beta}$ , we know, since  $s^* < 1$ , that the first-order condition with respect to  $s$  in (3.2) must hold:

$$\begin{aligned} \frac{\partial H_s(x_1, 0)}{\partial s} &= -x_1 f'(sx_1) \{ \Pi_2[c - f(x_1), c - f(sx_1)] \\ &\quad + \Pi_1[c - f(sx_1), c - f(x_1)] \} = 0. \end{aligned}$$

Evaluating the second partial derivative of  $H_s$  w.r.t.  $s$

$$\begin{aligned} \frac{\partial^2 H_s(x_1, 0)}{\partial s^2} &= x_1^2 [f'^2(sx_1) \{ \Pi_{22}[c - f(x_1), c - f(sx_1)] \\ &\quad + \Pi_{11}[c - f(sx_1), c - f(x_1)] \} \\ &\quad - f''(sx_1) \{ \Pi_2[c - f(x_1), c - f(sx_1)] \\ &\quad + \Pi_1[c - f(sx_1), c - f(x_1)] \}] \end{aligned}$$

along the solution to the first-order condition yields

$$\begin{aligned} \left[ \frac{\partial^2 H_s(x_1, 0)}{\partial s^2} \right]_* &= x_1^2 f'^2(sx_1) \{ \Pi_{22}[c - f(x_1), c - f(sx_1)] \\ &\quad + \Pi_{11}[c - f(sx_1), c - f(x_1)] \} > 0 \end{aligned}$$

by assumption (3.4). This shows that  $H_s(x_1, 0)$  is strictly quasi-convex in  $s$ , so that  $s^* = \underline{\beta}$  or  $s^* = 1$ . Since the latter is not the case (from part (a)), we must have  $s^* = \underline{\beta}$ .  $\square$

**Proof of Corollary 3.9.** Follows directly from Proposition 3.8.  $\square$

**Proof of Proposition 3.10.** With the (further) change of variable

$$\begin{cases} y_1 \doteq f(z_1), & \text{or} \\ y_2 \doteq f(z_2), & \end{cases} \quad \begin{cases} z_1 = g(y_1), & (\text{with } g \doteq f^{-1}), \\ z_2 = g(y_2) & \end{cases}$$

the firms' joint objective (6.11) can be rewritten as

$$G(y_1, y_2) \doteq \Pi(c - y_1, c - y_2) + \Pi(c - y_2, c - y_1) - \frac{g(y_1) + g(y_2)}{1+s} \quad (6.12)$$

on  $\Delta'_s = \{(y_1, y_2) : g(y_1) \geq sg(y_2) \text{ and } g(y_2) \geq sg(y_1)\}$ .

It can be verified that  $G$  is jointly strongly concave in  $(y_1, y_2)$  on  $\Delta'_s$  for every  $s \in [0, 1]$ , as long as (3.5) holds: to do so, simply check that (3.5) leads to  $0 > G_{11}, |G_{11}| > G_{12}, 0 > G_{22}, |G_{22}| > G_{12}$ , for all  $s \in [0, 1]$ , and hence also to  $G_{11}G_{22} \geq G_{12}^2$ . We claim then that any argmax  $(y_1^*, y_2^*)$  of  $G$  must have  $y_1^* = y_2^*$ . For otherwise, if  $(a, b)$  is an argmax of  $G$  on  $\Delta'_s$  with  $a \neq b$ , then symmetry of  $G$  and  $\Delta'_s$  in  $(y_1, y_2)$  implies that  $(b, a)$  must also be an argmax, in which case strict concavity of  $G$  leads to  $((a+b)/2, (a+b)/2) \in \Delta'_s$  yielding a higher value of  $G$  than the presumed maximum, a contradiction. Hence every argmax of  $G$  is on the diagonal. By Lemma 3.4, this implies that  $H_s^*$  is strictly increasing in  $s$  (in the original decision variables), and thus  $s = 1$  or case  $J$  is the optimal R&D cartel.  $\square$

**Proof of Proposition 4.1.** With the change of variable

$$z_1 = \sqrt{\frac{2}{\gamma}(x_1 + sx_2)} \quad \text{and} \quad z_2 = \sqrt{\frac{2}{\gamma}(x_2 + sx_1)},$$

the objective function in (4.1) becomes

$$G_s(z_1, z_2) = \frac{1}{9}(a - c + 2z_1 - z_2)^2 + \frac{1}{9}(a - c + 2z_2 - z_1)^2 - \frac{\gamma}{2(1+s)}(z_1^2 + z_2^2),$$

and the constraints  $x_1 \geq 0$  and  $x_2 \geq 0$  become  $z_1 \geq \sqrt{s}z_2$  and  $z_2 \geq \sqrt{s}z_1$ .<sup>16</sup> Hence the optimal R&D cartel solves

$$\max_{s \in [0, 1], z_1, z_2} G_s(z_1, z_2) \quad \text{subject to } z_1 \geq \sqrt{s}z_2 \text{ and } z_2 \geq \sqrt{s}z_1.$$

Note that if  $(s^*, z_1^*, z_2^*)$  is an optimal R&D cartel, then either  $z_1^* > 0$  or  $z_2^* > 0$  since

$$\left. \frac{\partial G_s(z, z)}{\partial z} \right|_{z=0} = \frac{4}{9}(a - c) > 0.$$

Let  $(s^*, z_1^*, z_2^*)$  be an optimal R&D cartel. We show that either  $s^* = 0$  or  $s^* = 1$ . Suppose to the contrary that  $s^* \in (0, 1)$ . Since either  $z_1^* > 0$  or  $z_2^* > 0$  then  $G_s(z_1^*, z_2^*)$  is strictly increasing in  $s$ . Furthermore, either  $z_1^* = \sqrt{s^*}z_2^*$  or  $z_2^* = \sqrt{s^*}z_1^*$ , since otherwise if both  $z_1^* > \sqrt{s^*}z_2^*$  and  $z_2^* > \sqrt{s^*}z_1^*$  then there is an  $s' > s^*$  such that  $z_1^* > \sqrt{s'}z_2^*$ ,  $z_2^* > \sqrt{s'}z_1^*$ , and  $G_{s^*}(z_1^*, z_2^*) > G_{s^*}(z_1^*, z_2^*)$ , contradicting that  $(s^*, z_1^*, z_2^*)$  is an optimal R&D cartel. (In terms of the original decision variables, if  $s^* \in (0, 1)$ , then one firm's R&D expenditure must be zero.)

Since  $G_s(z_1, z_2)$  is symmetric in  $(z_1, z_2)$ , we can assume without loss of generality that  $z_1^* = \sqrt{s^*}z_2^*$ . Define  $k^* = z_1^* + z_2^*$  to be the sum of the effective R&D's in the optimal R&D cartel. Then  $z_1^* = k^*\sqrt{s^*}/(1 + \sqrt{s^*})$  and  $z_2^* = k^*/(1 + \sqrt{s^*})$ . If  $(s^*, z_1^*, z_2^*)$  is an optimal R&D cartel, then  $s^* \in \arg \max_{s \in [0, 1]} G_s(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))$ . To see this, suppose to the contrary that  $s^* \notin \arg \max_{s \in [0, 1]} G_s(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))$ . Then there is an  $s' \in [0, 1]$  such that  $G_{s^*}(k^*\sqrt{s'}/(1 + \sqrt{s'}), k^*/(1 + \sqrt{s'})) > G_{s^*}(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))$ . Denote  $k^*\sqrt{s'}/(1 + \sqrt{s'})$  by  $z'_1$  and  $k^*/(1 + \sqrt{s'})$  by  $z'_2$ , it's easy to see that  $z'_1 \geq \sqrt{s'}z'_2$  and  $z'_2 \geq \sqrt{s'}z'_1$ , and hence that  $(s', z'_1, z'_2)$  is a feasible R&D cartel. This contradicts that  $(s^*, z_1^*, z_2^*)$  is an optimal R&D cartel. Note that increasing  $s$  in  $G_s(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))$  corresponds to increasing  $z_1$ , while holding  $z_1 = \sqrt{s}z_2$  and holding  $z_1 + z_2$  fixed.

Proof of (i): We show that if  $s^* \in \arg \max_{s \in [0, 1]} G_s(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))$ , then either  $s^* = 0$  or  $s^* = 1$ , and hence the optimal R&D cartel has either full or no spillovers. Differentiating  $G_s(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))$  with respect to  $s$  yields

$$\frac{\partial G_s(k^*\sqrt{s}/(1 + \sqrt{s}), k^*/(1 + \sqrt{s}))}{\partial s} = \frac{1}{2}(k^*)^2 \frac{\gamma - 2(1 - \sqrt{s})}{\sqrt{s}(1 + \sqrt{s})^3}. \quad (6.13)$$

<sup>16</sup> We have that

$$x_1 = \frac{\gamma}{2} \frac{z_1^2 - sz_2^2}{1 - s^2} \quad \text{and} \quad x_2 = \frac{\gamma}{2} \frac{z_2^2 - sz_1^2}{1 - s^2} \quad \text{for } s < 1.$$

For  $s = 1$  we have that  $z_1 = z_2 = \sqrt{2(x_1 + x_2)/\gamma}$ , which implies  $x_1 + x_2 = \frac{\gamma}{2}z_1^2 = \frac{\gamma}{2}z_2^2$ .

Suppose that  $s^*$  is an interior critical point of  $G_s(k^*\sqrt{s}/(1+\sqrt{s}), k^*/(1+\sqrt{s}))$ . Then  $s^*$  satisfies  $\gamma - 2(1-\sqrt{s^*}) = 0$  by (6.13). A straightforward calculation shows that

$$\frac{\partial^2 G_s(k^*\sqrt{s}/(1+\sqrt{s}), k^*/(1+\sqrt{s}))}{\partial s^2} = \frac{1}{4}(k^*)^2 \frac{8\sqrt{s}-6s-\gamma-4\gamma\sqrt{s}+2}{(\sqrt{s})^3(1+\sqrt{s})^4}.$$

Evaluating this expression at  $s = s^*$  and replacing  $\gamma$  in the expression above with  $2(1-\sqrt{s^*})$ , we obtain

$$\left. \frac{\partial^2 G_s(k^*\sqrt{s}/(1+\sqrt{s}), k^*/(1+\sqrt{s}))}{\partial s^2} \right|_{s=s^*} = \frac{1}{2}(k^*)^2 \frac{1}{s^*(1+\sqrt{s^*})^3} > 0.$$

Hence any interior critical point is a local minimum, and so  $s^* \in \arg \max_{s \in [0,1]} G_s(k^*\sqrt{s}/(1+\sqrt{s}), k^*/(1+\sqrt{s}))$  implies  $s^* = 0$  or  $s^* = 1$ .

Proof of (ii): If  $\gamma > 2$ , then

$$\frac{\partial G_s(k^*\sqrt{s}/(1+\sqrt{s}), k^*/(1+\sqrt{s}))}{\partial s} = \frac{1}{2}(k^*)^2 \frac{\gamma - 2(1-\sqrt{s})}{\sqrt{s}(1+\sqrt{s})^3} > 0$$

for  $s > 0$  and hence the optimal R&D cartel has  $s^* = 1$ .

We now show that  $\frac{a}{c} > \frac{5}{2}$  implies  $s^* = 1$ . By part (i) either  $s^* = 0$  or  $s^* = 1$ , and hence if  $(s^*, z_1^*, z_2^*)$  is an optimal R&D cartel, then either

$$(z_1^*, z_2^*) \in \arg \max_{z_1, z_2} G_0(z_1, z_2) \quad \text{subject to } z_1 \geq 0 \text{ and } z_2 \geq 0$$

or

$$(z_1^*, z_2^*) \in \arg \max_{z_1, z_2} G_1(z_1, z_2) \quad \text{subject to } z_1 \geq z_2 \text{ and } z_2 \geq z_1.$$

We show that if  $(\hat{z}_1, \hat{z}_2) \in \arg \max_{z_1, z_2} G_0(z_1, z_2)$  subject to  $z_1 \geq 0$  and  $z_2 \geq 0$ , and if  $(\bar{z}_1, \bar{z}_2) \in \arg \max_{z_1, z_2} G_1(z_1, z_2)$  subject to  $z_1 \geq z_2$  and  $z_2 \geq z_1$  then  $G_0(\hat{z}_1, \hat{z}_2) < G_1(\bar{z}_1, \bar{z}_2)$ , and hence  $s^* = 1$ .

Assume without loss of generality that  $\hat{z}_1 \geq \hat{z}_2$ . If  $\hat{z}_1 = \hat{z}_2 > 0$  then clearly  $G_0(\hat{z}_1, \hat{z}_2) < G_1(\hat{z}_1, \hat{z}_2) \leq G_1(\bar{z}_1, \bar{z}_2)$ . If  $\hat{z}_1 = \hat{z}_2 = 0$ , then  $G_0(\hat{z}_1, \hat{z}_2) = G_1(\hat{z}_1, \hat{z}_2) < G_1(\bar{z}_1, \bar{z}_2)$  since

$$\left. \frac{dG_1(z, z)}{dz} \right|_{z=0} = \frac{4}{9}(a-c) > 0.$$

We now consider the case where  $\hat{z}_1 > \hat{z}_2$ . We have that

$$\begin{aligned} G_0(\hat{z}_1, \hat{z}_2) &\leq \frac{1}{9}(a-c+2\hat{z}_1-\hat{z}_2)^2 + \frac{1}{9}(a-c+2\hat{z}_2-\hat{z}_1)^2 - \frac{\gamma}{2}\hat{z}_1^2 \\ &= \frac{2}{9}(a-c+\hat{z}_1)^2 - \frac{\gamma}{2}\hat{z}_1^2 - \frac{1}{9}(\hat{z}_1-\hat{z}_2)(2a-2c-3\hat{z}_1+5\hat{z}_2) \\ &\leq \frac{2}{9}(a-c+\hat{z}_1)^2 - \frac{\gamma}{2}\hat{z}_1^2 - \frac{1}{9}(\hat{z}_1-\hat{z}_2)(2a-2c-3c) \\ &< \frac{2}{9}(a-c+\hat{z}_1)^2 - \frac{\gamma}{2}\hat{z}_1^2 \leq G_1(\bar{z}_1, \bar{z}_2), \end{aligned}$$

where the first inequality follows from the definition of  $G_0(\hat{z}_1, \hat{z}_2)$  and  $\frac{\gamma}{2}\hat{z}_2^2 \geq 0$ , the second inequality follows from  $\hat{z}_1 \leq c$  and  $\hat{z}_2 \geq 0$ , the strict equality follows from  $2a > 5c$  and

$\hat{z}_1 > \hat{z}_2$ , and the last inequality follows from the definition of  $G_1(\bar{z}_1, \bar{z}_2)$  and the constraint when  $s = 1$  that  $z_1 = z_2$ .

Proof of (iii): By part (i) either  $s^* = 0$  or  $s^* = 1$ . We show that  $9\gamma < 4\frac{a}{c}$  and  $\frac{a}{c} < \frac{5}{2}$  implies  $\max_{z_1, z_2 \in [0, c]} G_0(z_1, z_2) > \max_{z_1, z_2 \in [0, c]} G_1(z_1, z_2)$ , i.e.,  $s^* = 0$ . Since  $9\gamma < 4\frac{a}{c}$  and  $\frac{a}{c} < \frac{5}{2}$ , then  $9\gamma < 10$  and hence the objective  $G_0(z_1, z_2)$  is jointly strictly convex in  $(z_1, z_2)$  and thus it is maximized on “corners” of  $[0, c]^2$ , i.e., its maximizers are among  $(c, 0), (0, c), (0, 0)$ , or  $(c, c)$ . We have  $G_0(c, 0) = G_0(0, c) = (a - c + 2c)^2/9 + (a - 2c)^2/9 - \frac{\gamma}{2}c^2$ . It is straightforward to show that if  $9\gamma \leq 4\frac{a}{c}$ , then the maximizers of  $G_1(z_1, z_2, 1)$  are  $(c, 0)$  and  $(0, c)$ . We have  $G_1(c, 0) = \frac{2}{9}a^2 - \frac{\gamma}{2}c^2$ . A simple calculation establishes that  $\frac{a}{c} < \frac{5}{2}$  implies  $G_0(c, 0) > G_1(c, 0)$ , and hence  $s^* = 0$ . Furthermore,  $G_0(z, z) < G_1(c, 0)$  for  $z = 0$  and  $z = c$  implies the optimal R&D cartel has  $(z_1^*, z_2^*) \in \{(c, 0), (0, c)\}$  or, in terms of the original decision variables,  $(x_1^*, x_2^*) \in \{(\frac{\gamma}{2}c^2, 0), (0, \frac{\gamma}{2}c^2)\}$ .  $\square$

## References

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