Games and Economic Behavior **31,** 1–25 (2000)
doi:10.1006/game.1999.0734, available online at http://www.idealibrary.com on IDEAL®

One-Way Spillovers, Endogenous Innovator/Imitator Roles, and Research Joint Ventures*

Rabah Amir

Department of Economics, Odense University, DK 5230 Odense M, Denmark
E-mail: raa@busieco.ou.dk

and

John Wooders

Department of Economics, University of Arizona, Tucson, Arizona 85721
E-mail: jwooders@bpa.arizona.edu

Received March 3, 1998

We consider a two-period duopoly characterized by a one-way spillover structure in process R&D and a very broad specification of product market competition. We show that *a priori* identical firms always engage in different levels of R&D, at equilibrium, thus giving rise to an innovator/imitator configuration and ending up with different sizes. We also provide a general analysis of the social benefits of, and firms' incentive for, forming research joint ventures. Another contribution is methodological, illustrating how submodularity (R&D decisions are strategic substitutes) can be exploited to provide a general analysis of a R&D game. *Journal of Economic Literature* Classification Numbers: C72, L13, O31.

Key Words: oligopolistic R&D; one-way spillovers; research joint ventures; submodularity.

1. INTRODUCTION

This paper develops a two-stage strategic model of process R&D/product market competition for an *ex-ante* symmetric duopoly under imperfect appropriability of R&D. In the model, at the first stage firms conduct pro-

* The authors thank Claude d'Aspremont, Kai-Uwe Kühn, Stanley Reynolds, Lars-Hendrik Röller, Bernard Sinclair-Desgagné, Peter Zemsky, and two anonymous referees, as well as seminar participants at C.O.R.E., INSEAD, the University of Arizona, the Free and Humboldt Universities, Berlin, the W.Z.B./C.E.P.R. Conference on Industrial Policy, and the 1st Conference on Industrial Organization at Carlos III—Madrid, for their comments and suggestions.

0899-8256/00 $35.00

cess R&D and at the second stage, upon observing the final unit costs, firms compete in the product market. R&D is imperfectly appropriable, with R&D spillovers flowing from the firm with higher R&D activity to its rival (but never vice-versa) in a binomial fashion: With probability β a full spillover occurs and with probability $(1-\beta)$ no spillover occurs. A uni-directional spillover structure, such as this one, is appropriate whenever there is a natural (complete) order to the cost-reducing innovations that can be achieved. In that case know-how may flow only from the firm further along in its R&D program to its less advanced rival.[1]

We derive two different sets of results. First, with a relatively wide scope of generality in the product market specification, we establish the existence of a subgame-perfect equilibrium, as well as a general characterization of its properties. In particular, we show that no equilibrium can be symmetric even though the two competing firms are *ex ante* identical. Thus an industry configuration endogenously emerges yielding an R&D innovator (the more R&D intensive firm) and an R&D imitator. Firms differ in size (which in a Cournot or Bertrand framework is tied to unit production cost) and in R&D intensity (which might involve R&D strategy, lab type and size, and the composition of R&D, were the R&D process explicitly modeled).

The second set of results concerns the performance comparison of a cartelized research joint venture (RJV) or joint lab (denoted Case J) and pure R&D competition (denoted Case N). In Case J, R&D is conducted in a jointly owned lab run at equal cost by the firms together, whereas in Case N each firm independently conducts R&D in its own lab. In both Case J and Case N firms behave noncooperatively in the product market. The performance criteria of interest are R&D propensity, consumer surplus, and producer surplus. Our main results here provide general conditions on the R&D cost function and the equilibrium second stage profit function which insure the superiority of the joint lab over R&D competition. These conditions boil down to demand being high enough relative to initial unit cost and/or R&D costs being convex enough, depending on the criterion of interest. A version of the latter condition, which d'Aspremont and Jacquemin (1988) refer to as a second-order condition, is found in every paper in the R&D literature.

The present paper also makes a methodological contribution to the analysis of two-stage games, in the context of a two-stage R&D game. The approach we introduce consists of representing the product market competition at the second stage by a function $\Pi(\cdot, \cdot)$, which gives the Nash profit to a firm as a function of the two post-R&D unit costs. An assumption

[1]In addition to its interpretation as the probability that a spillover occurs, the spillover parameter β can also be interpreted in novel ways as an inverse measure of patent length, or of imitation lag: See Amir and Wooders (1999) for details.

fundamental to our analysis is that Π is submodular. This assumption, as well as the other basic assumptions of our analysis, are satisfied under the common specifications of Cournot competition, with either differentiated or homogenous products, and Bertrand competition with sufficiently differentiated products. Therefore our analysis provides a unified treatment of strategic R&D encompassing most specifications of product market competition considered in previous related work. Throughout the present paper an ancillary objective is to employ only minimally sufficient assumptions on Π needed for our results, thereby preserving as high a level of generality as possible.

The submodularity of Π is inherited by the overall payoff function of the two-stage game (thus making R&D decisions strategic substitutes, a natural property).[2] This allows us to apply recent powerful results from the theory of supermodular[3] games in order to establish the existence of a pure strategy subgame-perfect equilibrium, to conduct the relevant comparative statics analysis, and to obtain general answers in the evaluation of R&D cooperation and competition. Standard existence results are not applicable since the one-way nature of spillovers leads to the two-stage payoff having a fundamental nonconcavity, independently of any curvature assumptions placed on the primitives of the model.

Our results relate to two different strands of literature, one on intra-industry heterogeneity and the other on R&D cooperation. In the literature on intra-industry heterogeneity Katz and Shapiro (1987) and Boyer and Moreaux (1997) also deal with R&D models, while Hermalin (1994) considers firms' internal control structures. These paper all develop *ex-ante* symmetric models with only asymmetric equilibria.[4] Related studies in evolutionary economics and management strategy have been around for a longer time: see Röller and Sinclair-Desgagné (1996) for a survey.

R&D cooperation has already been investigated by Katz (1986), d'Aspremont and Jacquemin (1988, 1990) (henceforth referred to as AJ) and Kamien *et al.* (1992) (henceforth KMZ). In these papers spillovers are multi-directional, the spillover parameter measuring the fraction of a firm's R&D activity that spills to each of its rivals (see Amir (1998) for a comparative critique). With linear demand and a general R&D cost function, KMZ establishes that Case J dominates Case N (as well as each of

[2]See Athey and Schmutzler (1995) for complementarities in R&D types in a one-firm model, and Bagwell and Staiger (1994) for a setting more related to ours.

[3]See Topkis (1978, 1979, 1998), Vives (1990), Milgrom and Roberts (1990a), Milgrom and Shannon (1994), and Amir (1996b).

[4]This contrasts with the literature on industry dynamics where exogenous idiosyncratic shocks lead to a heterogeneous distribution of characteristics among perfectly competitive firms: see Jovanovic (1982), Lambson (1992), Hopenhayn (1992), and Flaherty (1980).

the other R&D scenarios they consider) for the performance criteria of interest.

The analysis of AJ, based on linear demand and a quadratic R&D cost function, has inspired numerous follow-up papers investigating various aspects of R&D cooperation.[5] Within the same specification as AJ, but with one-way spillovers, Amir and Wooders (1999) provide a complete characterization that nicely complements the present analysis.

The paper is organized as follows: in Section 2 we introduce our model of R&D competition with one-way spillovers and analyze its equilibria. In Section 3 we compare R&D competition and RJV cartelization. Proofs of our results, as well as a brief summary of some (simplified) results on submodular games needed here, are provided in the Appendix.

2. THE NONCOOPERATIVE MODEL

2.1. *The Model*

Consider an industry composed of two *a priori* identical firms, each with initial unit cost c, engaged in a two-stage game. In the first stage, Firms 1 and 2 decide on unit cost reduction x and y, with $x, y \in [0, c]$, on the basis of a known R&D cost schedule $f(\cdot)$. In the second stage, upon observing the new unit costs, the firms compete in the product market by choosing outputs (i.e., Cournot competition) or prices (i.e., Bertrand competition). There is no need to specify the mode of competition as the equilibrium profits of the second stage are modeled by a general function which has both models as special cases.

While this two-stage framework is standard in the recent R&D literature, our set-up departs from previous ones in the way imperfect appropriability of R&D is modelled. We consider R&D processes where leakages flow only from the more R&D-active firm to the rival in an all-or-nothing probabilistic fashion. Specifically, given autonomous cost reductions by Firms 1 and 2 of x and y, respectively, with (say) $x \geq y$, the effective (or final) cost reductions are given by X and Y, respectively, with

$$X = x \text{ and } Y = \begin{cases} x \text{ with probability } \beta \\ y \text{ with probability } 1 - \beta. \end{cases} \tag{2.1}$$

This spillover process is a natural one in a number of different contexts.[6] If there is a complete order to the cost-reducing innovations that can be

[5]See Henriques (1990), DeBondt *et al.* (1992), and Suzumura (1992), among numerous others. See also Amir and Wooders (1998).

[6]Another reasonable spillover process is the deterministic analog, defined by $X = x$ and $Y = y + \beta(x - y) = \beta x + (1 - \beta)y$. Unfortunately, payoffs in the two-stage game are not submodular for large β under this process, as the reader can easily verify.

undertaken, then only the more R&D-active firm can generate spillovers. Alternatively, if there are many different R&D programs that a firm can undertake, but R&D programs are unrelated, then β can be interpreted as the probability that the less R&D active of the two firms is successful in imitating its rival.[7] See Amir and Wooders (1999) for a detailed discussion of these and other interpretations.

We restrict attention to subgame-perfect equilibria. A (pure) strategy for Firm i is a pair (x_i, a_i) where $x_i \in [0, c]$ is firm i's autonomous cost reduction and a_i: $[0, c]^2 \to \mathbb{R}$ is a map from profiles of post-R&D unit costs to the set of product market decisions (outputs or prices). The overall payoff to a firm is simply its second-stage profit minus its first-stage R&D cost.

The following basic assumptions are in effect throughout the paper.

(A1) For every pair of R&D decisions $(x, y) \in [0, c]^2$, the second-stage (product market) game has a unique Nash equilibrium, with corresponding payoffs (i.e., profits) given by a function Π of the two firms' post R&D unit costs. Here, $\Pi(\cdot, \cdot)$ denotes the Nash profits of the firm whose unit cost is the first argument.

(A2) (i) Π: $[0, c]^2 \to \mathbb{R}$ is continuous, and strictly submodular.

(ii) Π is nonincreasing (nondecreasing) in its first (second) argument.

(iii) $\Pi(c_1, c_1) < \Pi(c_2, c_2)$ if $c_1 > c_2$.

(A3) f is nondecreasing and $f(0) \geq 0$.

Some of our results require the following smoothness assumption (with Part (ii) of (A4) being a minor strengthening of (A2)(iii) given (A2)(ii)).[8]

(A4) (i) Π and f are twice continuously differentiable.

(ii) $|\Pi_1(z, z)| > |\Pi_2(z, z)|$, for all $z \in [0, c]$.

A major innovation in the present paper is our treatment of the product market competition, which we now argue offers a broad scope of generality in at least two different situations. The first situation is the standard one, whereby the second stage represents a one-shot game in product market decisions. In this context, Assumptions (A1)–(A4) may be justified and interpreted as follows. The equilibrium uniqueness assumption (A1) is convenient, if somewhat restrictive. For the Cournot model, for instance, Amir

[7]When R&D programs are inter-related, each firm can benefit from the R&D activity of its rival, and one of the standard multi-way deterministic spillover processes would be more adequate (see AJ and KMZ).

[8]Indeed, as (A2)(iii) says that $\Pi(z, z)$ is decreasing in z, we have (with smoothness) that $\Pi_1(z, z) + \Pi_2(z, z) \leq 0$, which is nearly the same as (A4)(ii).

(1996b) shows that it holds whenever $P(\cdot) - c_i$ is a log-concave function, where $P(\cdot)$ is the inverse demand function and c_i is the unit cost of Firm i, $i = 1, 2$. This is implied, in particular, by $P(\cdot)$ itself being log-concave, and is thus more general than most studies using the Cournot model. For the Bertrand model with differentiated products, Milgrom and Roberts (1990a) give a uniqueness argument for a variety of commonly used specifications.

(A2)(i) may be viewed as a (negative) complementarity condition as it holds that the improvement in a firm's profit resulting from a drop in own costs increases with the cost of the rival firm. (A2)(ii) is self-explanatory: a firm's profit decreases with own cost, but increases with rival's cost. (A2)(iii) says that in a symmetric duopoly, a unit drop in both firms' costs raises their profits. Put differently, own cost effects dominate rival's cost effects on profit. While reasonable, this assumption does impose restrictions on demand in a Cournot setting: see Seade (1985) or Kimmel (1992) for a detailed treatment of this issue. Finally, (A3) is clearly a natural assumption.

We now discuss the generality of our approach. First, observe that Assumptions (A1)–(A4) are all satisfied in the commonly adopted cases of

(i) Cournot competition with linear demand $P(Q) = a - bQ$ and unit costs k_1, k_2, which leads to equilibrium profits given by $\Pi(k_1, k_2) = (a - 2k_1 + k_2)^2/9b$, and

(ii) Bertrand competition with differentiated products, linear demand $q_i = a - p_i + bp_j$, $0 < b < 1$, $i, j = 1, 2$, $i \neq j$, and units costs k_1, k_2, which leads to equilibrium profits equal to $\Pi(k_1,\ k_2) = [(2 + b)a - (2 - b^2)k_1 + bk_2]^2$.

Other examples which can easily be verified as satisfying (A1)–(A4) include Cournot competition with a quadratic demand function $P(Q) = a - bQ^2$, $Q \leq \sqrt{a/b}$, which leads to equilibrium profits equal to $\Pi(k_1, k_2) = \frac{1}{16}(2a + k_2 - 3k_1)^2/\sqrt{b(2a - k_1 - k_2)}$, or Cournot competition with linear demand and differentiated products. On the other hand, with hyperbolic demand $P(Q) = (Q + 1)^{-1}$ it can be checked (see Mirman et al., 1994) that (A2)(i) fails to hold. Nonetheless, since most studies in the R&D literature are based on linear demand, the present treatment constitutes a major improvement in generality. Another desirable feature of this approach is its unifying power; it dispenses with the usual distinctions between Cournot and Bertrand competition, and homogeneous and differentiated products, thus allowing a comprehensive treatment of the effects of process-R&D on market competition.

The second possible situation to which the two-stage model at hand can be applied is a novel one made possible by our approach. Here, Π may represent the overall equilibrium payoff to a multi-stage game, possibly with an infinite horizon, and in several product markets simultaneously.

In this perspective, the R&D decision is a long-term choice followed by many short-term (possibly inter-related) market decisions. The latter may be prices, output levels, or even some combination of the two. This new interpretation enhances the level of realism captured by the two-stage model, since in practice (i) production technology remains fixed over relatively long intervals of time, while market decisions are typically made much more frequently, and (ii) firms compete over several markets. In this context, Assumption (A1) may be overly restrictive, owing to repeated-game effects. (A1) could then be replaced by an assumption that equilibrium in product market subgames, following the application of some meaningful criterion (e.g., Pareto optimality, renegotiation proofness, etc.), is uniquely selected.

Although our assumptions are substantially more general than in previous R&D studies, Assumption (A2)(i) remains less than fully justified in the sense that minimally sufficient conditions on the demand and cost functions of a duopoly for (A2)(i) to hold are not known.[9] Nonetheless, our approach emerges as the most general among those that can yield answers to the questions of interest in this paper, without making *ad-hoc* assumptions (such as the existence of subgame-perfect equilibria).[10]

We now complete the description of the two-stage game by deriving the payoff functions. Observe that the game is symmetric, i.e., independent of a relabeling of the players. The expected payoff to Firm 1 (say) is, with x, $y \in [0, c]$,

$$F(x, y) = \begin{cases} \beta\Pi(c - x, c - x) + (1 - \beta)\Pi(c - x, c - y) - f(x) \overset{\circ}{=} U(x, y) \\ \qquad \text{if } x \geq y \\ \beta\Pi(c - y, c - y) + (1 - \beta)\Pi(c - x, c - y) - f(x) \overset{\circ}{=} L(x, y) \\ \qquad \text{if } x \leq y. \end{cases} \tag{2.2}$$

The expected payoff to Firm 2, defined similarly, is given by $F(y, x)$, in view of the symmetry of the game. The expressions in (2.2) reflect the facts that the firms get (i) the same second-stage profits corresponding to the larger cost reduction for both, with probability β, (ii) the profits corresponding to

[9]Topkis (1978) proved that optimization preserves supermodularity (of the value function with respect to the remaining variable). What is needed here is a generalization of this result to equilibrium payoffs in a game.

[10]Some previous studies in the related literature on two-stage R&D games have also proposed general models. Both Brander and Spencer (1983) and Suzumura (1992) analyze models with second-stage Cournot competition under very general assumptions on the demand and cost functions. However, neither paper addresses the issue of existence of subgame-perfect equilibrium.

their autonomous cost reductions with probability $(1-\beta)$, and (iii) pay for their autonomous cost reduction only.

It is easy to see that F inherits the continuity property of Π (from Assumption (A2)(i)). It turns out that F also inherits the submodularity of Π (also in (A2)(i)), but not the differentiability of Π and f, which fails along the diagonal of $[0, c]^2$, or the concavity of each line in (2.2) assumed below for some of our results. This is intended only as a preview here and will be established later.

2.2. *Properties of the Noncooperative R&D Model*

Here, we state and interpret our results for the two-stage game under consideration. Since the second-stage game admits a unique Nash equilibrium, every Nash equilibrium (x^*, y^*) of the game with payoffs (2.2) induces a subgame-perfect equilibrium of the two-stage game, and vice-versa. In view of this one-to-one correspondence, we use the two terminologies interchangeably.

We begin with the fundamental property of the game at hand (strategic substitutability), and a key structural characteristic of its equilibria (asymmetry).

THEOREM 2.1. *Assume* (A1)–(A2) *hold. Then the following are true*:

(i) *The game with payoffs* (2.2) *is submodular, and hence has a pure-strategy Nash equilibrium.*

(ii) *Every interior pure-strategy Nash equilibrium is asymmetric if* (A4) *holds and* $\beta > 0$.

(iii) *Every pure-strategy Nash equilibrium is asymmetric if, in addition to the hypothesis of* (ii), *the following holds* (*here subscripts denote partial derivative*) :

$$f'(0) < -\beta\Pi_2(c, c) - \Pi_1(c, c) \quad \textit{and} \quad f'(c) > -(1-\beta)\Pi_1(0, 0). \tag{2.3}$$

A discussion of these results is provided at the end of this subsection. The next result deals with uniqueness of equilibrium. Observe that in view of the symmetry of the game and the fact that no equilibrium can involve the firms taking the same decisions (Theorem 2.1), the sharpest uniqueness result would yield two equilibria.

THEOREM 2.2. *Under Assumptions* (A1)–(A4) *and* (2.3), *the R&D game* (*with payoffs* (2.2)) *has exactly two pure-strategy Nash equilibria, of the form* $(\bar{x}, \underline{x})$ *and* $(\underline{x}, \bar{x})$, *with* (*say*) $\bar{x} > \underline{x}$, *if in addition* $\beta \in (0, 1]$ *and*

$$f''(\cdot) > \big(\Pi_{11} - \Pi_{12}\big)[c-(\cdot), z], \quad \forall z \in [0, c] \tag{2.4}$$

and

$$f''(\cdot) > \big(\Pi_{11} + 2\Pi_{12} + \Pi_{22}\big)[c-(\cdot), c-(\cdot)]. \tag{2.5}$$

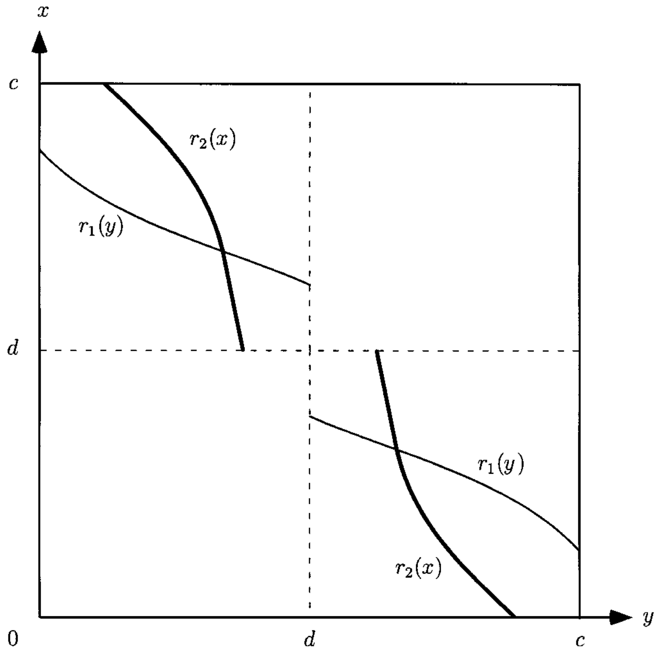

FIGURE 1

Define the best-response correspondence in the usual way, i.e., (say) for Firm 1, $r_1(y) = \arg\max\{F(x, y)\colon x \in [0, c]\}$. The next result holds that r_1 and r_2 are essentially as depicted in Figure 1 (note that $r_1 = r_2$, by symmetry).

LEMMA 2.3. *Under the hypothesis of Theorem* 2.2 (*i.e.*, (A1)–(A4), (2.3)–(2.5)), r_1 *and* r_2 *are continuous nonincreasing functions everywhere in* $[0, c]$ *except at one point* $d \in (0, c)$ *where* $r_i(d^-) > d > r_i(d^+)$.

The last results in this section deal with comparative statics of the equilibrium $(\bar{x}, \underline{x})$ as the probability of a spillover increases in $(0, 1]$.

THEOREM 2.4. *Under* (A1) *and* (A2), *the following hold as* β *increases in* $(0, 1]$:

(i) *Holding one firm's R&D level constant, the extremal best-responses of the other firm are nonincreasing* (*in other words*, r_1 *and* r_2 *shift down*).

(ii) *The total equilibrium R&D level* $\bar{x} + \underline{x}$ (*associated with the unique Nash equilibrium pair* $(\bar{x}, \underline{x})$ *and* $(\underline{x}, \bar{x})$) *decreases if, in addition*, (2.4) *and* (2.5) *hold*.

(iii) $\bar{x}$ *itself decreases if, in addition,* (2.4) *and* (2.5) *and the following hold*:

$$f''(\cdot) \geq (1-\beta)\Bigg[\Pi_{11}(c-(\cdot), c-z) + \frac{\Pi_1(c-(\cdot), c-z)\Pi_{12}(c-z, c-(\cdot))}{(\Pi_1+\Pi_2)(c-z, c-z) - \Pi_1(c-z, c-(\cdot))}\Bigg], \quad \forall z > (\cdot), \quad \forall \beta. \tag{2.6}$$

To gain insight into the nature of Conditions (2.3)–(2.6), it is instructive to consider them under the well-known cases of (a) Cournot competition with linear demand and homogeneous goods, and (b) Bertrand competition with linear demand and differentiated goods, both with R&D cost function $f(x) = \frac{\gamma}{2}x^2$. Leaving out the computational details we note that (i) the first part of (2.3) always holds for both (a) and (b), (ii) the second part of (2.3) becomes $9b\gamma > 4(1-\beta)\frac{a}{c}$ for Case (a) and $\gamma > 2(1-\beta)(2-b^2)(2+b)\frac{a}{c}$ for Case (b), (iii) (2.4) becomes $9b\gamma > 12$ for Case (a) and $\gamma > (b+2)^2(1-b)^2$ for Case (b). Finally, a sufficient condition can be derived for (2.6): $9\gamma > 16(1-\beta)$ for Case (a), which is shown in Amir and Wooders (1999) to be rather tight, and $\gamma \geq 4(1-\beta)(2-b^2)^2$ for Case (b). It is worthwhile to observe that all these seemingly different conditions boil down to some common qualitative requirements: that R&D costs be sufficiently convex, and/or market size be sufficiently small (relative to cost), and/or spillovers or extent of product homogeneity be sufficiently high. Conditions similar to γ being large enough have been used in all related work following AJ (who refer to them as second-order conditions).

We now provide a discussion of the results of this section. Theorem 2.1(i) suggests that our model is well defined under remarkably general conditions. In particular, the absence of any concavity assumptions on the payoff function F is new in the R&D literature. For the overall payoff function F to inherit the submodularity property of the equilibrium profit function Π, Assumption (A2)(iii) is crucial. Submodularity of F here has the usual negative complementarity interpretation: The marginal returns to increasing R&D expenditures decrease with the rival's R&D expenditure, and this holds independently of whether the firm is receiving or giving away spillovers!

In view of the asymmetry of equilibria, our model is a natural candidate for explaining the ubiquitous inter-firm heterogeneity within most industries. The driving force behind this endogenous firm heterogeneity is the anticipation of (probabilistic) R&D spillovers from the leading firm to its rival. Under such a spillover structure, firms endogenously emerge with different production cost structures through the very process of adopting (costly) technological progress. Thus, the competing firms end up with different levels of R&D activity (hence with different types of R&D strategy/labs), different firm sizes, and different market shares in the product market.

Theorem 2.2 is a convenient result as it allows for more straightforward analysis of equilibrium behavior, unencumbered by the difficulties associated with multiple equilibria. For instance, it is needed for clear-cut answers to the comparative statics analysis of Theorem 2.4. Both theorems require assumptions on f'', which translate into strong convexity of f since Π is typically convex in own costs or even jointly. Similar assumptions have always been made in related studies (e.g., AJ, KMZ), and are needed to insure that payoffs are concave in own R&D decision. In our model, such assumptions can only yield concavity of each line in (2.2), but not of F itself.

While Theorems 2.4(i) and (ii) are intuitively clear, (iii) is perhaps less so. The fact that each firm would decrease its R&D level as β increases, holding the rival's R&D level constant, does not imply that, at equilibrium, both R&D levels go down.[11] In other words, there are two effects governing the response of $\bar{x}$ (say) to changes in β. The first is captured in 2.4(i) and is rather intuitive: The leading firm (or innovator) cuts down on R&D as the likelihood of full spillover to the rival increases, with the rival's R&D level constant. However, if the rival also decreases his R&D level, the other effect is that the firm under consideration will want to respond by increasing R&D activity. The overall effect on $\bar{x}$ then depends on the relative strength of these two effects. Condition (2.6) is needed to shift the balance towards a decline of $\bar{x}$, i.e., towards the first effect. To see this, observe that the f'' on the LHS of (2.6) refers to the imitator's cost function, the intuition being that if the latter's second derivative is large, $\underline{x}$ does not respond much to changes in β and in $\bar{x}$, so that the first effect above is the dominant one.

Finally, since the payoffs are continuous, the game at hand has a symmetric mixed-strategy equilibrium in R&D decisions. Since the game is supermodular, the support of the mixed strategies at equilibrium would be $[\underline{x}, \overline{x}]$. Under such a solution, the firms would still end up (endogenously) different with positive probability.

3. RESEARCH JOINT VENTURES

We consider here different R&D cooperation schemes among firms which remain competitors in the product market. These schemes are char-

[11]In the language of supermodularity analysis, one cannot find orders on the two actions sets that would make each payoff supermodular in the two decisions and in the pair (own decision, β). Hence, the comparative statics result for supermodular games cannot be invoked (Milgrom and Roberts, 1990a; Sobel, 1988).

acterized by two key features: whether firms coordinate in choosing R&D expenditure (i.e., "collude" in the first-stage of the game), and whether firms cooperate in the actual conduct of R&D (by increasing β).

Here, we are mainly concerned with only one RJV scenario: the joint lab. This is characterized by the firms running one joint R&D facility at half the cost each, and will be denoted by J. We note below that J is equivalent (for our model) to KMZ's case CJ, or cartelized RJV, whereby firms coordinate R&D expenditures in the first-stage and fully communicate during the R&D process (i.e., set the spillover rate equal to 1).

In the course of investigating the properties of Case J, it turns out that it is useful to also consider the following broader RJV specification. Let C_s denote the scenario whereby firms coordinate their R&D investments (so as to maximize total profits), while the spillover parameter is given by $s \in [0, 1]$. Thus, in particular, $s = 0, \beta, 1$ stand for the cases where the spillover rate is reduced to 0, kept as it is, and increased to 1 (its maximum value), respectively.[12] Note that the case $s < \beta$ is not necessarily economically meaningful within the context of our model in the sense that spillovers are generally thought of as being unpreventable by the firms. Nonetheless, the case $s = 0$ is particularly useful below for comparative purposes.

The joint objective function of the two firms in Case C_s (assuming w.l.o.g. that $x \geq y$) is to maximize $F(x, y) + F(y, x)$ over x, y in $[0, c]$, with β set equal to s, which reduces to

$$2s\Pi(c - x, c - x) + (1 - s)\big[\Pi(c - x, c - y) + \Pi(c - y, c - x)\big] - f(x) - f(y). \tag{3.1}$$

The single-firm objective in Case J is to maximize over $x \in [0, c]$

$$\Pi(c - x, c - x) - \tfrac{1}{2} f(x). \tag{3.2}$$

Observe that (3.1) reflects the (potential) operation of two separate R&D labs by the cartel, with variable spillover parameter, while (3.2) reflects the operation of one joint lab with equal cost sharing. In Case J a symmetric outcome necessarily obtains. As will be seen below, this may or may not be true for Case C_s, $s \in [0, 1)$. In both cases, the two firms compete in the product market, as captured by Π.

Our central concern in this section is a performance comparison between the noncooperative model of Section 2 (to be denoted N) and Case J (which we show below to be essentially equivalent to Case C_1). The performance criteria of interest here are: propensity for R&D, firm profits,

[12] The case C_β here is clearly the analog of the second scenario analyzed in AJ. See Salant and Shaffer (1998) on the emergence of asymmetry in Case C_β.

consumer and social welfare. The cases C_0 and C_β are analyzed here only as useful intermediate steps in the overall analysis.

We first point out that Cases J and C_1 are interchangeable in the following sense.

LEMMA 3.1. *Cases J and C_1 are equivalent in the sense that they both lead to the same optimal R&D levels and the same optimal total profits.*

It is still convenient to have the two cases as Case J is more readily interpretable and has symmetry built into it, while Case C_1 is useful below through its properties as the limit case of C_s as $s \to 1$.

Our first comparison of Cases J and N concerns R&D propensities. This requires, however, an intermediate lemma which is of independent interest, a comparison between Case J and Case N with $\beta = 0$ (the latter is denoted N_0 below). In dealing with this comparison, an additional assumption is now introduced as a new version of (A4). It quantifies the dependence of profits on own versus cross cost reductions in a symmetric duopoly setting.

(A5) Π and f are twice continuously differentiable and $|\Pi_1(z, z)| \geq 2|\Pi_2(z, z)|$, for all $z \in [0, c]$.

Clearly, (A5) is a stronger version of (A4). It is easily seen to be satisfied under Cournot competition with linear demand, with strict inequality if products are differentiated and with equality for homogeneous products. For Bertrand competition with differentiated products, (A5) can be seen to hold if and only if the cross-demand coefficient (denoted by b in the discussion of (A1)–(A3) in Section 2) is in the interval $(0, \sqrt{3} - 1] \approx (0, .73]$,[13] i.e., as long as demand is somewhat away from the well-known case of homogenous products ($b = 1$).[14]

LEMMA 3.2. *Under Assumptions* (A1)–(A3), (A5) *and* (2.4), *we have*

(i) *In Case N_0, there is a unique and symmetric equilibrium (x_0, x_0).*

(ii) *The equilibrium R&D level of Case J, x_J, satisfies $x_J \geq x_0$.*

We are now ready for the comparison of R&D propensities and profits (interpretations of the results are given later on).

[13]In their treatment of Bertrand competition, KMZ give $\frac{2}{3}$ as a *lower* bound for this critical value of b. Since our model and theirs are equivalent when $\beta = 0$ and demand is linear, the fact that our bound is sharper indicates that (A5) is tight (see also the proof of Lemma 3.2(ii)).

[14]It can easily be seen that the Π function corresponding to the case $b = 1$, given by

$$\Pi(c_1, c_2) = \begin{cases} (c_2 - c_1)D(c_2) & \text{if } c_1 < c_2 \\ 0 & \text{if } c_1 \geq c_2, \end{cases}$$

(where $D(\cdot)$ is the demand function) is not submodular in (c_1, c_2). Hence this case fails Assumption (A2) anyway, and thus does not fit our model.

PROPOSITION 3.3. *Under Assumptions* (A1)–(A3), *we have*

(i) $x_J \geq \bar{x} \geq \underline{x}$ (*with strict inequality whenever* $\beta > 0$) *if* (A5) *and* (2.4)–(2.6) *hold*.

(ii) *Total equilibrium profits are higher in Case J than in Case N, provided that at least one of the following conditions holds*:

$$2\Pi(c_2, c_2) \geq \Pi(c_1, c_2) + \Pi(c_2, c_1), \quad \textit{for all } c_1 \geq c_2. \tag{3.3}$$

$$f''(\cdot) > (\Pi_{11} - \Pi_{12})(c - (\cdot), z) + (\Pi_{22} - \Pi_{12})(z, c - (\cdot)), \quad \textit{for all } z \in [0, c]. \tag{3.4}$$

Due to the asymmetric nature of the equilibria in Case N, single-firm profit comparisons do not seem possible at this level of generality. Of course, even if total profits improve through cooperation, an asymmetric outcome in Case N also means firms have different incentives to cooperate in R&D, although transfers could be made so that both firms benefit.

The welfare comparison essentially follows from Proposition 3.3 once the following plausible assumption about consumer surplus is added (note that given the level of generality of the product market competition here, consumer surplus cannot be explicitly defined in the usual way).

(A6) Consumer surplus is decreasing in the firms' unit costs.

This assumption holds in most commonly used specifications of Cournot and Bertrand competition. In particular, it holds for the cases of linear demand reported in Section 2. For Cournot competition (with homogeneous products), it actually holds for any demand function, provided production costs are linear and a Cournot equilibrium exists (see Amir (1996b) for exact conditions). This is because total equilibrium output and price depend only on total unit cost (Bergstrom and Varian, 1985).

COROLLARY 3.4. *Regardless of whether full or no spillover is realized*, (*ex-post*) *social welfare is higher under Case J than under Case N, assuming* (A1)–(A3), (A5), (A6), (2.4)–(2.6), *and either* (3.3) *or* (3.4).

Next, we discuss our assumptions in the context of Bertrand and Cournot competition. Condition (2.6) was already discussed in Section 2.3. Note that requiring $x_J \geq \bar{x}$ is clearly stronger than requiring $2x_J \geq (1+\beta)\bar{x} + (1-\beta)\underline{x}$, i.e., that expected total cost reduction is higher in Case J than in Case N.

Condition (3.3) or (3.4) is needed to guarantee that total profits are higher for Case C_1 than for Case C_0 which, in turn, ensures that profits in Case J exceed profits in Case N. Condition (3.3), rewritten as $\Pi(c_2, c_2) - \Pi(c_1, c_2) \geq \Pi(c_2, c_1) - \Pi(c_2, c_2)$, says that effects on own profits of *any* discrete change in own cost exceeds those due to the same change in rival's cost, starting from a symmetric duopoly. Thus (3.3) strengthens (A4)(ii) which says the same thing but only for infinitesimal changes. Under Cournot competition with linear demand condition (3.3) is equivalent to $2a + 3c_2 - 5c_1 \geq 0$ (with $c_2 \leq c_1$). Since $c_2 \geq 0$ and $c_1 \leq c$, a sufficient condition for (3.3) in this case is $2a \geq 5c$. Thus (3.3) amounts to requiring demand to be high (relative to costs).

Condition (3.4) is needed to ensure concavity of the joint objective in Case C_0, therefore resulting in a symmetric R&D choice for this case. Hence (3.4) works by removing the asymmetry bias captured in Lemma 3.5 (see below). When symmetry prevails in Case C_0, the cartelized firms prefer full to no spillover. Under Cournot competition with linear demand Condition (3.4) is equivalent to $9\gamma > 18$, and hence requires that the R&D cost function be sufficiently convex. Both (3.3) and (3.4) have analogous interpretations for Bertrand competition.

We now provide a discussion of the results of this section emphasizing their relationship to related work on RJVs, namely, AJ and KMZ. As discussed in the Introduction, one motivation of the present paper is a re-examination of the principal conclusion from related work: that a joint lab or cartelized RJV dominates R&D competition in terms of equilibrium prices (and thus consumer welfare), firm profits, and hence social welfare. The reasons for questioning the validity of this conclusion in the present context are (i) the lack of generality of the previous analyses, (ii) the new spillover process introduced, and (iii) the fact that Case J yields symmetric outcomes as a built-in feature while Case N always leads to asymmetric equilibria. This last feature is important since, in typical specifications of Cournot and Bertrand competition (see examples in Section 2.1), the Π function is jointly convex and thus firms (jointly) prefer not to compete on equal terms in the product market, as we now show.

LEMMA 3.5. *Let Π be jointly convex on $[0, c]^2$, $k > 0$ and consider the following objective* (*with constraint*):

$$\{\Pi(c_1, c_2) + \Pi(c_2, c_1) \colon c_1 + c_2 = k\}. \tag{3.5}$$

Then the $\arg\max$ *of* (3.5) *consists of* $(0, k)$ *and* $(k, 0)$, *while the* $\arg\min$ *is* $(\frac{k}{2}, \frac{k}{2})$.

Roughly, the main finding here is that, in the present context, the principal conclusion of the RJV literature crucially requires new assumptions, i.e., (2.6) and (3.3) or (3.4), to ensure its validity.

Finally, observe that for Case N_0, our model is equivalent to AJ's and KMZ's. Thus, our Lemma 3.2(ii) may be viewed as a generalization of their analogous result to a broader class of profit functions (instead of that corresponding to linear demand).

APPENDIX

Summary of Submodular Optimization/Games

Here, we define all the notation and state all the results from submodular optimization needed in our analysis, in the simplest (but self-contained) form. Let I_1, I_2 be compact real intervals and F: $I_1 \times I_2 \to \mathbb{R}$.

F is submodular [strictly submodular] if for all $x_1 > x_2$ in I_1 and all $y_1 > y_2$ in I_2, we have $F(x_1, y_1) - F(x_1, y_2) \leq [<] F(x_2, y_1) - F(x_2, y_2)$. The following result is a special case of Topkis's Monotonicity Theorem (Topkis, 1978).

THEOREM 4.1. *If F is continuous in y and submodular* [*strictly submodular*] *in* (x, y), *then* $\arg\max_{y \in I_2} F(x, y)$ *has maximal and minimal* [*all of its*] *selections nonincreasing in* $x \in I_1$.

The next result identifies an easy test for submodularity, and is often called Topkis's Characterization Theorem:

THEOREM 4.2. *If F is twice continuously differentiable, F is submodular iff* $F_{12}(x, y) = (\partial^2 F(x, y)/\partial x \partial y \leq 0$. *Furthermore,* $F_{12}(x, y) < 0$ *implies strict submodularity.*

Finally, we need the following definition and existence result. A two-player game is submodular if both payoff functions are submodular and both action spaces are compact real intervals.

THEOREM 4.3. *A two-player submodular game has a pure strategy Nash equilibrium.*

Topkis (1979) proved this result for (n-player) supermodular games (F is supermodular iff $-F$ is submodular). Vives (1990) extended it to *two-player* submodular games. See also Milgrom and Roberts (1990a). Theorem 4.3 is not valid in general for games with more than two players.

Proofs

This section provides all the proofs for the results given in the previous sections, in the order given. We begin with some notation. Let $\Delta_u \overset{\circ}{=} \{(x, y) \in [0, c]^2\colon x \geq y\}$, $\Delta_l \overset{\circ}{=} \{(x, y) \in [0, c]^2\colon x \leq y\}$. (Note that, contrary to usual practice, x is along the vertical axis while y is on the horizontal axis below). With U and L as given by (2.2), by symmetry, Firm 2's payoff is $F(y, x) = L(y, x)$ if $y \leq x$, and $U(y, x)$ if $y \geq x$.

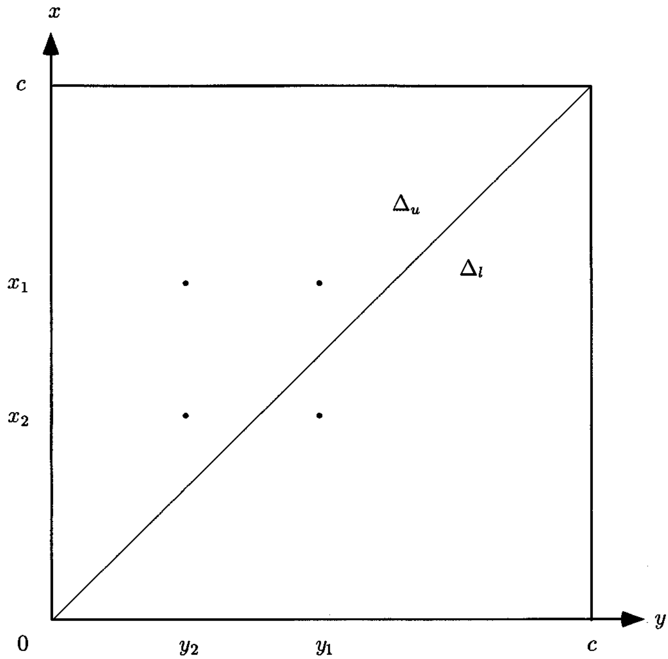

FIGURE 2

Proof of Theorem 2.1. (i) We show that F, as given by (2.2), is strictly submodular in (x, y).[15] To this end, fix x_1, x_2, y_1, y_2 in $[0, c]$ with $x_1 > x_2$, $y_1 > y_2$. If all four points (x_1, y_1), (x_1, y_2), (x_2, y_1), and (x_2, y_2) lie in Δ_u or in Δ_l, strict submodularity of F follows directly from the strict submodularity of Π (i.e., Assumption (A2)(i)), since only the middle term of U and L depends on both x and y.

If some of the four points lie in Δ_u and the rest in Δ_l, it is easily seen that there are four different cases. It turns out that the proofs of strict submodularity of F are all similar, so we present the case depicted in Fig. 2; i.e., (x_1, y_1), (x_1, y_2), (x_2, y_2) are in Δ_u while (x_2, y_1) is in Δ_l. We must then show that $U(x_1, y_1) - U(x_1, y_2) < L(x_2, y_1) - U(x_2, y_2)$. We clearly have, given the location of the points, $x_2 < y_1$. Hence, by Assumption (A.2)(iii),

$$0 \leq \beta\Pi(c - y_1, c - y_1) - \beta\Pi(c - x_2, c - x_2). \tag{4.1}$$

[15]Note that Topkis's Characterization Theorem cannot be used here, since F is not differentiable along the diagonal in $[0, c]^2$, but it can be used in the interior of Δ_u and Δ_l separately. On the other hand, Theorem 2* of Milgrom and Roberts (1990b) is an alternative approach to this proof.

Since Π is strictly submodular and f only depends on one variable,

$$\begin{aligned}\left[(1-\beta)\Pi(c-x_1, c-y_1) - f(x_1)\right] &- \left[(1-\beta)\Pi(c-x_1, c-y_2) - f(x_1)\right] \\ \leq \left[(1-\beta)\Pi(c-x_2, c-y_1) - f(x_2)\right] & \\ - \left[(1-\beta)\Pi(c-x_2, c-y_2) - f(x_2)\right]. & \qquad (4.2)\end{aligned}$$

Adding up (4.1), (4.2) and the trivial equality $\beta\Pi(c-x_1, c-x_1) - \beta\Pi(c-x_1, c-x_1) = 0$ and rearranging terms yields

$$\begin{aligned}&\left[\beta\Pi(c-x_1, c-x_1) + (1-\beta)\Pi(c-x_1, c-y_1) - f(x_1)\right] \\ &\quad - \left[\beta\Pi(c-x_1, c-x_1) + (1-\beta)\Pi(c-x_1, c-y_2) - f(x_1)\right] \\ &< \left[\beta\Pi(c-y_1, c-y_1) + (1-\beta)\Pi(c-x_2, c-y_1) - f(x_2)\right] \\ &\quad - \left[\beta\Pi(c-x_2, c-x_2) + (1-\beta)\Pi(c-x_2, c-y_2) - f(x_2)\right],\end{aligned}$$

which says that F is strictly submodular for the four-point choice of Fig. 2. This last inequality is strict since (4.1) is strict if $\beta > 0$ and (4.2) is strict if $\beta < 1$.

The argument for each of the remaining choices is similar, and thus left to the reader. Existence of a pure-strategy Nash equilibrium follows from Theorem 4.3.

(ii) Partial differentiation w.r.t. x yields

$$\begin{aligned}U_1(x, y) = -\beta\left[\Pi_1(c-x, c-x) + \Pi_2(c-x, c-x)\right] \\ - (1-\beta)\Pi_1(c-x, c-y) - f'(x)\end{aligned}$$

and $L_1(x, y) = -(1-\beta)\Pi_1(c-x, c-y) - f'(x)$. Along the diagonal $x = y$, the difference between these partials is $U_1(x, x) - L_1(x, x) = -\beta[\Pi_2(c-x, c-x) + \Pi_1(c-x, c-x)] > 0$ (by (A4)(ii)). This implies that x can never be a best response to x, for any $x \in (0, c)$, since a necessary condition for that is $U_1(x, x) \leq L_1(x, x)$.[16] Hence no interior equilibrium can be symmetric.

(iii) In view of (ii), it remains to show that $(0, 0)$ and (c, c) cannot be equilibria. To this end, consider $U_1(0, 0) = -\Pi_1(c, c) - \beta\Pi_2(c, c) - f'(0) > 0$ by (2.3), and $L_1(c, c) = -(1-\beta)\Pi_1(0, 0) - f'(c) < 0$ by (2.3). This implies that neither 0 nor c can be a best response to itself. ∎

Proof of Theorem 2.2. Since the game is symmetric, $(a, b) \in [0, c]^2$ is a Nash equilibrium whenever (b, a) is. Here, we show that there is exactly one such pair of equilibria. We first show that r_1, r_2 are as in Fig. 1.

[16]This last inequality is simply a generalized first-order condition for a maximum in the absence of differentiability of F.

It is easily checked that (2.4) and (2.5) imply that U and L are strictly concave in x (for fixed y), on Δ_u and Δ_l, respectively. Hence, if $r_1(\cdot)$, say, is discontinuous at some point y_0, then $r_1(y_0^-)$ and $r_1(y_0^+)$ cannot both lie in Δ_u or both in Δ_l (note here that r_1 is an upper semi-continuous correspondence, due to the joint continuity of F, so that $r_1(y_0^-)$ and $r_1(y_0^+)$ are both in $r_1(y_0)$). Furthermore, by Theorems 4.1 and 2.1(i), every selection from r_1 is nonincreasing. Also, by Theorem 2.1, r_1 cannot intersect the 45° line. Therefore, there exists a unique point $d \in (0, c)$ such that (i) r_1 is discontinuous at d, with $r_1(d^-) > d > r_1(d^+)$, i.e., $r_1(d^-) \in \Delta_u$ and $r_1(d^+) \in \Delta_l$, (ii) r_1 is continuous and lies in Δ_u for $y \in [0, d]$, and (iii) r_1 is continuous and lies in Δ_l for $y \in [d, c]$. In other words, r_1 and r_2 are essentially as depicted in Fig. 1.

Next, we show that there is a unique equilibrium in the rectangle $R := \{(x, y)\colon 0 \le x \le d \text{ and } d \le y \le c\} \subset \Delta_l$. We do this by showing that r_1 and r_2 are (essentially) contractions in R. Whenever r_1 is interior, the first-order condition $L_1[r_1(y), y] = 0$, the Implicit Function Theorem, and (A4) yield that r_1 is differentiable in R and

$$r_1'(y) = -\frac{L_{12}[r_1(y), y]}{L_{11}[r_1(y), y]} = \frac{(1-\beta)\Pi_{12}[c - r_1(y), c - y]}{f''[r_1(y)] - (1-\beta)\Pi_{11}[c - r_1(y), c - y]}.$$

Similarly, on R, $r_2'(x) = -U_{21}[x, r_2(x)]/U_{22}[x, r_2(x)]$, and thus

$$r_2'(x) = \frac{(1-\beta)\Pi_{12}[c - r_2(x), c - x]}{\Big(f''[r_2(x)] - (1-\beta)\Pi_{11}[c - r_2(x), c - x] - \beta(\Pi_{11} + 2\Pi_{12} + \Pi_{22})\,[c - r_2(x), c - r_2(x)]\Big)}.$$

Straightforward computations show that $r_1'(y) > -1$ iff

$$f''[r_1(y)] > (1-\beta)(\Pi_{11} - \Pi_{12})[c - r_1(y), c - y] \tag{4.3}$$

and $r_2'(x) > -1$ iff

$$f''[r_2(x)] > (1-\beta)(\Pi_{11} - \Pi_{12})\big[c - r_2(x), c - x\big] + \beta(\Pi_{11} + 2\Pi_{12} + \Pi_{22})\big[c - r_2(x), c - r_2(x)\big]. \tag{4.4}$$

Clearly, (2.4) and (2.5) imply (4.3) and (4.4), and hence also imply that $r_i'(\cdot) > -1$, $i = 1, 2$. Recapitulating, we have $r_i'(\cdot) \in (-1, 0]$ in the interior of R, $i = 1, 2$. Since $r_1(d^+) < d$ and r_1 is nonincreasing, whenever r_1 is not interior in R, it must be that $r_1 \equiv 0$. Hence $r_i'(\cdot) \in (-1, 0]$ in (all of) R. Then, uniqueness of equilibrium in R follows from a well-known argument (for a proof, see, e.g., Amir (1996a), Lemma 2.3).

By symmetry, there must be exactly two Nash equilibria of the form $(\bar{x}, \underline{x})$ and $(\bar{x}, \underline{x})$. ■

Proof of Lemma 2.3. This has already been proved in the first part of the proof of Theorem 2.2. ■

Proof of Theorem 2.4. (i) Here, we want to show that r_1 and r_2 shift down as β increases. To this end, we need to show that each payoff is submodular in own decision and β (holding the rival's decision constant), and then invoke (Topkis's) Theorem 4.1. In view of the symmetry of the game and Lemma 2.3, it suffices to show submodularity in R, i.e. (in view of Theorem 1), $L_{1\beta}(x, y) \leq 0$ and $U_{1\beta}(y, x) \leq 0$. We have $L_{1\beta}(x, y) = \Pi_1(c-x, c-y) \leq 0$, by (A2)(ii), and

$$\begin{aligned} U_{1\beta}(y, x) &= -\Pi_1(c-y, c-y) - \Pi_2(c-y, c-y) + \Pi_1(c-y, c-x) \\ &\leq -\Pi_2(c-y, c-y) \leq 0, \end{aligned}$$

where the first inequality follows from (A2)(i) and the fact that $y > x$ on R, and the second inequality follows from (A2)(ii). This completes the proof of Part (i).

(ii) Since the lines of constant $x+y$ have slope -1, the line $x+y = \underline{x} + \bar{x}$ lies between the graphs of r_1 and r_2 (and intersects them at $(\underline{x}, \bar{x})$). As β increases, and both r_1 and r_2 shift down (by Part (i)), it is easy to see that $\underline{x} + \bar{x}$ has to decrease too.

(iii) Consider the (unique) equilibrium $(\underline{x}, \bar{x})$ in R. If $(\underline{x}, \bar{x})$ is not interior in R, we know from the (last part of) the proof of Theorem 2.2 that it must be the case that $\underline{x} = 0$. Then the fact that $\bar{x}$ decreases in β follows directly from the fact that r_2 shifts down (as β increases), i.e., Theorem 2.4(i).

If $(\underline{x}, \bar{x})$ is interior in R, the following first-order conditions must hold:

$$\begin{aligned} &-\beta\left[\Pi_1(c-\bar{x}, c-\bar{x}) + \Pi_2(c-\bar{x}, c-\bar{x})\right] \\ &\qquad - (1-\beta)\Pi_1\left(c-\bar{x}, c-\underline{x}\right) - f'(\bar{x}) = 0, \end{aligned}$$

and

$$-(1-\beta)\Pi_1(c-\underline{x}, c-\bar{x}) - f'(\underline{x}) = 0.$$

Totally differentiating w.r.t. β, and collecting terms yields

$$\begin{aligned} &\left[\beta(\Pi_{11} + 2\Pi_{12} + \Pi_{22})(c-\bar{x}, c-\bar{x}) + (1-\beta)\Pi_{11}(c-\bar{x}, c-\underline{x}) - f''(\bar{x})\right]\frac{d\bar{x}}{d\beta} \\ &\quad + (1-\beta)\Pi_{12}(c-\bar{x}, c-\underline{x})\frac{d\underline{x}}{d\beta} \\ &\quad = (\Pi_1 + \Pi_2)(c-\bar{x}, c-\bar{x}) - \Pi_1(c-\bar{x}, c-\underline{x}) \end{aligned}$$

and

$$(1-\beta)\Pi_{12}(c-\underline{x}, c-\bar{x})\frac{d\bar{x}}{d\beta} + \left[(1-\beta)\Pi_{11}(c-\underline{x}, c-\bar{x}) - f''(\underline{x})\right]\frac{d\underline{x}}{d\beta}$$
$$= -\Pi_1(c-\underline{x}, c-\bar{x}).$$

Solving for $\frac{d\bar{x}}{d\beta}$ (e.g., using Cramer's rule), we get $\frac{d\bar{x}}{d\beta} \geq 0$ iff

$$f''(\underline{x}) \geq (1-\beta)\Bigg[\Pi_{11}(c-\underline{x}, c-\bar{x}) + \frac{\Pi_1(c-\underline{x}, c-\bar{x})\Pi_{12}(c-\bar{x}, c-\underline{x})}{(\Pi_1+\Pi_2)(c-\bar{x}, c-\bar{x}) - \Pi_1(c-\bar{x}, c-\underline{x})}\Bigg],$$

which is clearly implied by (2.6). ■

Proof of Lemma 3.1. Obvious, hence omitted. ■

Proof of Lemma 3.2. (i) In the game N_0, the payoff function of Firm 1 (say) is

$$\Pi(c-x, c-y) - f(x). \tag{4.5}$$

This game is clearly submodular as a consequence of (A2)(i). Hence, it has a Nash equilibrium. Uniqueness follows from the proof of Theorem 2.2 since (2.4) is the same as (4.3) with $\beta = 0$. In other words, uniqueness follows here from the best response having slopes in $(-1, 0]$ as shown before (with $\beta = 0$). Finally, symmetry of the unique equilibrium in $[0, c]^2$ follows from the fact that the payoff (4.5) is strictly concave in x (implied by (2.4)), thus leading to continuous best-response functions which intersect at the 45° line.

(ii) Proceed by contradiction and assume that $x_J < x_0$. Assuming x_J and x_0 are both interior, they satisfy the following first-order conditions:

$$-2(\Pi_1+\Pi_2)(c-x_J, c-x_J) - f'(x_J) = 0 \tag{4.6}$$

and

$$-\Pi_1(c-x_0, c-x_0) - f'(x_0) = 0. \tag{4.7}$$

By (A2)(ii) and (A5), $\Pi_1(c-x_J, c-x_J) + 2\Pi_2(c-x_J, c-x_J) \leq 0$. Summing up this inequality and (4.6) yields

$$\begin{aligned} &-\Pi_1(c-x_J, c-x_J) - f'(x_J) \\ &\quad \leq 0 = -\Pi_1(c-x_0, c-x_0) - f'(x_0), \quad \text{by (4.7)} \\ &\quad \leq -\Pi_1(c-x_0, c-x_J) - f'(x_0), \end{aligned}$$

where the last inequality follows from (A2)(i) and the contradiction hypothesis $x_J < x_0$. Now, the first and the last terms in the string of inequalities above give the derivative of $\Pi(c-(\cdot), c-x_J) - f(\cdot)$ evaluated at x_J and x_0 respectively. Since $x_0 > x_J$, this clearly contradicts the concavity of $\Pi(c-(\cdot), c-x_J) - f(\cdot)$ which is itself implied by (2.4), the submodularity of Π, and Theorem 4.2.

Without interiority, the only cases that might cause any difficulty are $x_0 = c$ and $x_J = 0$ (since we are trying to show that $x_J \geq x_0$). First, we show that if $x_0 = c$, then $x_J = c$ too. By (A2), $-\Pi_1(c-x, c-x) \geq -\Pi_1(c-x, 0)$, for all $x \in [0, c]$. Also, by (A5), $-\Pi_1(c-x, c-x) - 2\Pi_2(c-x, c-x) \geq 0$. Adding up the two inequalities yields $-2\Pi_1(c-x, c-x) - 2\Pi_2(c-x, c-x) - f'(x) \geq -\Pi_1(c-x, 0) - f'(x)$, which says that the derivative with respect to x of the objective of

$$\max\left\{2\Pi(c-x, c-x) - f(x) - f(y) : x, y \in [0, c]\right\} \tag{4.8}$$

is always higher than that of (4.5) with $y = c$. Since, as in the previous paragraph, $\Pi(c-(\cdot), 0) - f(\cdot)$ is concave by (2.4), $x_0 = c = \arg\max\{\Pi(c-(\cdot), 0) - f(\cdot)\}$ implies that the latter maximand is nondecreasing. Hence, so is (4.8) since it has a larger derivative $\forall\, x$. Hence $x_J = c$ too.

Next, we show that $x_J = 0$ implies $x_0 = 0$. If $x_J = 0$, (4.6) becomes $-2\Pi_1(c, c) - 2\Pi_2(c, c) - f'(0) \leq 0$. By (A5), $\Pi_1(c, c) + 2\Pi_2(c, c) \leq 0$. Adding up yields $-\Pi_1(c, c) - f'(0) \leq 0$. Since $\Pi(c-(\cdot), c) - f(\cdot)$ is concave by (2.4), we have $x_0 = 0$. ■

Proof of Proposition 3.3. (i) For extra clarity here, let us index the R&D equilibrium of Section 2 by the associated value of β, i.e., write $\bar{x}_\beta$ for $\bar{x}$ and $\underline{x}_\beta$ for $\underline{x}$, for all $\beta \in (0, 1]$. For $\beta = 0$, we have $\bar{x}_0 = \underline{x}_0 = x_0$ (from Lemma 3.2).

From Theorem 2.4(iii), we know that $\bar{x}_\beta < \bar{x}_0 = x_0$, for all $\beta \in (0, 1]$. Hence, from Lemma 3.2, $x_J \geq x_0 > \bar{x}_\beta$. This completes the proof of Proposition 3.3(i).

(ii) We first show that (3.3) is sufficient for the conclusion of this Proposition. To this end, note that in Case C_β, the Nash equilibrium $(\bar{x}, \underline{x})$ is a feasible joint decision. Hence, equilibrium profits are no lower in Case C_β than in Case N. Next rewrite the joint objective (3.1), assuming w.l.o.g. that $x \geq y$, as

$$\begin{aligned}
&\Pi(c-x, c-y) + \Pi(c-y, c-x)\\
&\quad + s\left[2\Pi(c-x, c-x) - \Pi(c-x, c-y) - \Pi(c-y, c-x)\right]\\
&\quad - f(x) - f(y).
\end{aligned} \tag{4.9}$$

By (3.3), this objective is nondecreasing in s, for fixed (x, y). Hence, optimal profits are higher for $s = 1$, i.e., for Case C_1 or equivalently (Lemma 3.1) for Case J, than for any other $s \in [0, 1]$, in particular $s = \beta$. Thus, profits are higher in Case J than in Case N.

We now show that (3.4) is also sufficient for the same conclusion. The joint objective for Case C_0 is (from (3.1) with $s = 0$)

$$G(x, y) \stackrel{\circ}{=} \Pi(c - x, c - y) + \Pi(c - y, c - x) - f(x) - f(y). \tag{4.10}$$

It can be verified that (4.10) is (jointly) strictly concave in (x, y) if (3.4) holds (to check this, one can see that $G_{11} > G_{12}$ and $G_{22} > G_{12}$ follow from (3.4)). Since (4.10) is also symmetric in (x, y), there must be a unique arg max, which is also symmetric, i.e., of the form (x^*, x^*). (Otherwise, if (a, b) is an arg max with $a \neq b$, then symmetry implies that (b, a) is also an arg max. With strict concavity, this leads to $(\frac{a+b}{2}, \frac{a+b}{2})$ yielding a strictly higher value than the max itself, a contradiction.)

Consequently, one can restrict the maximization of (4.10) to choices on the diagonal, i.e., replace (4.10) with $\max_x\{2\Pi(c - x, c - x) - 2f(x)\}$, which is clearly below the joint objective in Case J, i.e. (from (3.2)), $2\Pi(c - x, c - x) - f(x)$. Hence, equilibrium profits are higher in Case J than in Case C_0. Equilibrium profits in Case C_s are convex in s since (i) the objective function for Case C_s is linear in s (see (4.9)), and (ii) the pointwise supremum of a collection of linear functions in s is convex in s (Rockafellar, 1970). Thus equilibrium profits are lower for Case C_β than for either C_0 or $C_1 \equiv J$. Altogether then, $C_1 \equiv J$ yields higher profits than all the C_s, $s \in [0, 1]$, and thus also higher than Case N (recall that the latter has lower profits than C_β). ■

Proof of Corollary 3.4. Since Proposition 3.3(ii) holds here, we know that producer welfare is higher in Case J than in Case N. By Proposition 3.3(i) we have $x_J \geq \bar{x} \geq \underline{x}$. The imitator's effective R&D level is $\underline{x}$ with probability $(1 - \beta)$ and $\bar{x}$ with probability β, and is hence always below x_J too. Hence, by (A6), consumer welfare is higher in Case J than in Case N, and thus so is total welfare. ■

Proof of Lemma 3.5. First, observe that both the objective and the constraint in (3.5) are symmetric in (c_1, c_2). Hence, if (a, b) is an optimizer, so is (b, a). Since the iso-profit curves are concave, the arg max must be a boundary choice. Therefore, $(k, 0)$ and $(0, k)$ must form the arg max. Analogous reasoning for the minimization case leads to the arg min being unique and equal to $(\frac{k}{2}, \frac{k}{2})$. ■

REFERENCES

Amir, R. (1996a). "Continuous Stochastic Games of Capital Accumulation with Convex Transitions," *Games Econ. Behav.* **15**, 111–131.

Amir, R. (1996b). "Cournot Oligopoly and the Theory of Supermodular Games," *Games Econ. Behav.* **15**, 132–148.

Amir, R. (1998). "Modelling Imperfectly Appropriable R&D via Spillovers," D.P. 98-01, Odense University.

Amir, R., and Wooders, J. (1999). "Effects of One-way Spillovers on Market Shares, Industry Price, Welfare, and R&D Cooperation," *J. Econ. Management Strategy* **8**, 223–249.

Amir, R., and Wooders, J. (1998). "Cooperation vs. Competition in R&D: the Role of Stability of Equilibrium," *J. Econ.* **67**, 63–73.

Athey, S., and Schmutzler, A. (1995). "Product and Process Flexibility in an Innovative Environment," *RAND J. Econ.* **26**, 557–574.

Bagwell, K., and Staiger, R. (1994). "The Sensitivity of Strategic and Corrective R&D Policy in Oligopolistic Industries," *J. Internat. Econ.* **36**, 133–150.

Bergstrom, T., and Varian, H. (1985). "When are Nash Equilibria Independent of the Distribution of Agents' Characteristics," *Rev. of Econ. Stud.* **52**, 715–718.

Boyer, M., and Moreaux, M. (1997). "Strategic Considerations in the Choice of Technological Flexibility," *J. Econ. Management Strategy* **6**, 347–376.

Brander, J., and Spencer, B. (1983). "International R&D Rivalry and Industrial Strategy," *Rev. Econ. Stud.* **50**, 707–722.

d'Aspremont, C., and Jacquemin, A. (1988). "Cooperative and Noncooperative R&D in Duopoly with Spillovers," *Am. Econ. Rev.* **78**, 1133–1137; "Erratum," *Amer. Econ. Rev.* **80**, 641–642.

DeBondt, R., Slaets, P., and Cassiman, B. (1992): "The Degree of Spillovers and the Number of Rivals for Maximum effective R&D," *Internat. J. Indust. Organiz.* **10**, 35–54.

Flaherty, T. (1980). "Industry Structure and Cost-Reducing Investment," *Econometrica* **48**, 1187–1209.

Henriques, I. (1990). "Cooperative and Noncooperative R&D in Duopoly with Spillovers: Comment," *Amer. Econ. Rev.* **80**, 638–640.

Hermalin, B. (1994). "Heterogeneity in Organizational Form: Why Otherwise Identical Firms Choose Different Incentives for their Managers," *RAND J. Econ.* **25**, 518–537.

Hopenhayn, H. (1992). "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica* **60**, 1127–1150.

Jovanovic, B. (1982). "Selection and the Evolution of Industry," *Econometrica* **50**, 649–670.

Kamien, M., Muller, E., and Zang, I. (1992). "Research Joint Ventures and R&D Cartels," *Amer. Econ. Rev.* **82**, 1293–1306.

Katz, M. (1986). "An Analysis of Cooperative Research and Development," *RAND J. Economics* **17**, 527–543.

Katz, M., and M. Shapiro (1987). "R&D Rivalry with Licensing or Imitation," *Amer. Econ. Rev.* **77**, 402–420.

Kimmel, S. (1992). "Effect of Cost Changes on Oligopolists' Profits," *J. Indust. Econ.* **40**, 441–449.

Lambson, V. (1992). "Competitive Profits in the Long-Run," *Rev. Econ. Stud.* **59**, 125–142.

Milgrom, P., and Roberts, J. (1990a). "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," *Econometrica* **58**, 1255–1278.

Milgrom, P., and Roberts, J. (1990b). "The Economics of Modern Manufacturing: Technology, Strategy, and Organization," *Amer. Econ. Rev.* **80**, 511–528.

Milgrom, P., and Shannon, C. (1994). "Monotone Comparative Statics," *Econometrica* **62**, 157–180.

Mirman, L., Samuelson, L., and Schlee, E. (1994). "Strategic Information Manipulation in Duopolies," *J. Econ. Theory* **62**, 363–384.

Rockafellar, T. (1970). *Convex Analysis*. Princeton, NJ: Princeton Univ. Press.

Röller, L.-H., and Sinclair-Desgagné, B. (1996). "On the Heterogeneity of Firms," *European Econ. Rev.* **40**, 531–539.

Salant, S., and Shaffer, G. (1998). "Optimal Asymmetric Strategies in Research Joint Ventures," *Internat. J. Indust. Organiz.* **16**, 195–208.

Seade, J. (1985). "Profitable Cost Increases and the Shifting of Taxation," University of Warwick Economic Research Paper #260.

Sobel, M. (1988). "Isotone Comparative Statics for Supermodular Games," mimeo, S.U.N.Y., Stony Brook.

Suzumura, K. (1992). "Cooperative and Noncooperative R&D with Spillovers in Oligopoly," *Amer. Econ. Rev.* **82**, 1307–1320.

Topkis, D. (1978). "Minimizing a Submodular Function on a Lattice," *Oper. Res.* **26**, 305–321.

Topkis, D. (1979). "Equilibrium Points in Nonzero Sum *n*-Person Submodular Games," *SIAM J. Control Optim.* **17**, 773–787.

Topkis, D. (1998). *Supermodularity and Complementarity*, Princeton, NJ: Princeton Univ. Press.

Vives, X. (1990). "Nash Equilibrium with Strategic Complementarities," *J. Math. Econ.* **19**, 305–321.